Analytical study of the dynamics of capillary discharge plasmas for recombination x-ray lasers using H-like ions

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A simple but appropriate analytical study has been made to investigate the dynamics of a capillary discharge-produced plasma, thus obtaining Z-scaling formulas for the experimental parameters for a collisional recombination pumped x-ray laser using H-like ions. The analysis has been found to be in an excellent agreement with the results from a series of magnetohydrodynamic (MHD) simulations both in the dynamics and in the prediction of the appearance of gain of the H_α line in the H-like ion for the elements with the atomic number $z=4-13$. The formulas are valuable because (1) they identify key physical processes and (2) serve as a guideline for the design of an experimental setup as well as the full MHD simulation and (3) provide an overview on the application of the capillary discharge to the recombination laser. The comparison of the experimental parameters for collisional recombination x-ray lasers with those for collisional excitation x-ray laser [Shlyaptsev *et al.* SPIE **2012**, 99(1994)] has been made also. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1515273]

I. INTRODUCTION

The amplification of extreme ultraviolet radiations has been actively pursued and successfully demonstrated in various ways during the last decade.[1] Most of the successful results came from the usage of high power optical lasers to produce adequate gain media. However the huge size of high-power optical lasers has been a block to the widespread usage of the demonstrated x-ray lasers.

In parallel, efforts have been made to develop compact x-ray lasers. Among them was a proposal of the Ni-like Mo XV (19.1 nm) scheme[2] and the experimental demonstration of the amplification of CVI 18.2 nm, and Al XI 15.4 nm using small-size lasers.[3,4] The recent development of fs laser technology not only allows one to construct a compact x-ray laser system[5] but also encourage one to propose a new x-ray laser scheme for fs x-ray lasers.[6,7] A capillary discharge, and a cylindrical discharge with a small radius of a few millimeters, has also been studied due to its compactness and efficient energy coupling. Using the collisional excitation pumping scheme (CEPS), the Ne-like Ar IX line at 46.9 nm was amplified up to saturation.[8,9] A table-top 46.9 nm laser was realized in the laboratory.[10] Using the collisional recombination pumping scheme (CRPS), the H-like C VI line at 18.2 nm (Ref. 11) and the Li-like O VI lines at 49.8 and 52.0 nm (Ref. 12) were amplified. Toward the successful amplification in the water-window region (2.3–4.4 nm) using a capillary discharge, the following questions need to be addressed:

(a) In what manner are experimental parameters (e.g., initial gas density, current pulse duration, maximum current) scaled with the atomic number in a capillary-discharge-produced plasma?
(b) Is it possible to get the amplification in the water window region using a capillary-discharge-produced plasma?

For the case of CEPS, a good amount of experimental data exists for the reasonable comparison with simulation results, based on which the optimum experimental parameters toward the amplification at 9 nm were obtained by magnetohydrodynamics (MHD) simulations.[13] However for the case of CRPS, the experimental data are not satisfactory to answer the above questions properly due to small gain[11] and short gain length.[12] Thus an extensive series of MHD simulations has been performed for carbon species, resulting in finding new, better experimental conditions for the H-like C VI H_α (18.2 nm) line.[14]

This experience motivated us to investigate the dynamics of a capillary discharge plasma for a H-like CRPS x-ray laser to answer the above questions in a proper manner. A MHD simulation study could also give answers; however, it requires tremendous effort and time to span all the possible parameters in order to have a good overlook on the dependence of the dynamics of a discharge on the experimental parameters such as initial current, current pulse duration, initial gas density, etc.

For a gas-discharged cylindrical plasma column, some efforts have been made to model its dynamics such as the snowplow model,[15] the snowplow energy model,[16] and the

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slug model.[17] The snowplow model predicts only the pinch time quite well. Miyamoto has included the effect of pressure, thus describing the motion of the plasma column before and after pinch; but the simultaneous agreement of both the pinch time and radius with a MHD simulation could not be achieved.[18] The slug model is capable of describing the shock front but its prediction of the pinch time and the after-pinch motion of the plasma column is quite different from the result of a MHD simulation.[18] The failure of these analytical models might originate from the usage of a single physical model to describe the axial discharge plasma. Since the driving forces for compression and expansion phases are different as a magnetic pressure and a thermal pressure, respectively, two phases should be considered in different ways.

The previous MHD simulation study of a carbon plasma revealed that the dynamics can be described by a shock implosion at pinch and a subsequent adiabatic expansion.[18] The population inversion between the $n=2$ and 3 levels of H-like C VI can be obtained once the expansion cooling is faster than or comparable to the recombination rate from fully stripped ions to H-like ions. Considering such dominant dynamics with the help of the snowplow model,[15] the analytic z-scaling formulas for the adequate experimental conditions for different atomic numbers were obtained.

In Sec. II, the derivation of the z-scaling formulas will be presented in detail. To confirm the z-scaling formulas and justify assumptions made in the derivation, MHD simulations were performed. The comparison between the MHD simulation and the analytical results, which shows a good agreement, will be presented in Sec. III. The z-scaling formulas were then used to investigate the experimental conditions for the formation of high gain with respect to z. The results will be also discussed in Sec. III, along with the comparison of the efficiency between CRPS and CEPS.

II. Z-SCALING

The realization of CRPS x-ray lasers using H-like ions requires the following two steps to occur in a right manner: the first one is the production of fully stripped ions and the second the dominant collisional recombination process of fully stripped ions into H-like ions to build a population inversion between the $n=2$ and $n=3$ levels in H-like ions. In the second step, it is important that the decrease of temperature take place at such a fast rate that the collisional recombination process from fully stripped ions to H-like ions becomes dominant among various recombination processes. In a capillary discharge plasma, the collisional ionization in a high temperature plasma at pinch plays the role of the first step and a subsequent adiabatic expansion can serve the second step under a certain condition.[18] Thus, in the analysis of a capillary discharge plasma for a CRPS x-ray laser, the consecutive dynamics of heating and cooling should be adequately described.

There are quite complicated phenomena involved in the dynamics of capillary discharge plasmas from atomic processes in a microscopic level to hydrodynamics such as shock propagation, propagation of ionization wave,[19] reversed current profile,[20] and so on in a macroscopic level. Even though it is impossible to describe all the details of dynamics analytically, the following overall dynamics can be treated in an analytic way. When a discharge begins, an induced azimuthal magnetic field compresses a cylindrical plasma column, producing finally a high density and temperature plasma at the time of maximum compression called pinch. Afterwards the plasma column begins to expand due to the high pressure of the plasma. But instabilities in such a plasma usually break the uniformity of the cylindrical column at pinch, producing hot spots which radiates copious hard x rays. For the application of a capillary plasma to the amplification of soft x-ray radiation, the cylindrical uniformity should be maintained, which has been experimentally achieved using a preionization current pulse[8,12] in a capillary channel prior to a main discharge pulse. Thus, in the theoretical analysis of the dynamics of a capillary discharge for a CRPS x-ray laser, what to be analyzed are then how and how much a temperature increases at pinch and how fast and how much it decreases in an expansion phase to build a population inversion between the H-like ionic levels of $n=2$ and 3.

For the description of the plasma dynamics, the snowplow model[15] is adopted for the pinch state and an adiabatic expansion for the expansion phase. The simplified dynamics are matched to the conditions for the generation of a high-degree of population inversion suggested by Elton.[21] The condition of the H-like L_α line being optically thin was also imposed to ensure the generation of gain in combination with the requirement of a fast decrease of temperature. This analysis leads to the z-scaling formulas for the adequate experimental parameters for achiving population inversion between $n=2$ and $n=3$ levels of H-like ions in a capillary discharge system, such as an initial radius (R_o), an initial density (N_o), the period (τ) of a sinusoidal discharge current pulse, and its peak current (I_o).

A. Pinch phase

Even though the snowplow model[15] is a simple one, it predicts the pinch time, τ_p, very well. The pinch time is described by

$$\tau_p = 2.5\times 10^{-3} A^{1/4} R_o N_o^{1/4} \left(\frac{\tau}{I_o}\right)^{1/2} \quad [\text{ns}], \tag{1}$$

$$\simeq 3.0\times 10^{-3} z^{1/4} R_o N_o^{1/4} \left(\frac{\tau}{I_o}\right)^{1/2} \quad [\text{ns}], \tag{2}$$

where A is the atomic mass number, R_o is an initial plasma radius in cm, N_o is an initial plasma density in cm^{-3}, τ is the period of a discharge current pulse in ns, and I_o is the peak current in kA. The approximate equation [Eq. (2)] is obtained with $A\simeq 2z$, where z is the atomic number. At pinch, the plasma temperature increases rapidly due to the thermalization of a shock implosion and an adiabatic compression but in a fast capillary discharge, the shock implosion is considered as a dominant heating process. Ohmic heating is a dominant heating process in a early compresssing phase and suggested to cause the propagation of an ionization wave,[19] but its contribution to the pinch temperature was found to be negligible in a series of MHD simulations.[18] The temperature is then approximated by

$$T_p \simeq \tfrac{1}{2} m_i v^2, \tag{3}$$

where m_i the ion mass and v the implosion velocity. The shock implosion heats ions not electrons but due to a rapid equilibration between ions and electrons, the above temperature can be considered as the electron temperature at pinch. The validation of this assumption will be discussed in Sec. III. Approximating the implosion velocity by $v = R_o/\tau_p$, with $A \simeq 2z$, the electron temperature at pinch, $T_{e,p}$, is expressed as

$$T_{e,p} \simeq 1.2\times 10^{11}\left(\frac{z}{N_o}\right)^{1/2}\left(\frac{I_o}{\tau}\right) \ [\mathrm{eV}]. \tag{4}$$

For CRPS x-ray lasers using H-like ions, this temperature should be sufficiently high to produce fully stripped ions. Taking this temperature to be a half of the ionization energy of a H-like ion, i.e.,

$$T_{e,p} \simeq 0.5\times 13.6 \ z^2 \ [\mathrm{eV}], \tag{5}$$

the following condition for the initial parameters are obtained from Eq. (4):

$$\left(\frac{I_o}{\tau N_o^{1/2}}\right) \simeq 5.7\times 10^{-11} \ z^{3/2}. \tag{6}$$

Actually, in a high density and transient plasma, rate equations need to be evaluated instead of above approximation [Eq. (5)], which makes the analysis very complicated. However, MHD simulations presented in the following section show that the above approximation works quite well.

The pinch radius, R_p, is difficult to be obtained from a simple physical analysis. A series of MHD simulations[14] tells us that the plasma radius decreases by an order of magnitude by pinch,

$$R_p \simeq 0.1 \ R_o. \tag{7}$$

Using the mass conservation, the ion density at pinch, $N_{i,p}$, is written by

$$N_{i,p} = \left(\frac{R_o}{R_p}\right)^2 N_o \simeq 10^2 \ N_o. \tag{8}$$

The electron density at pinch is then written by

$$N_{e,p} \simeq 10^2 \ zN_o, \tag{9}$$

under the assumption of sufficient fully stripped ions at pinch.

B. Expansion phase

It has been shown that a fast adiabatic expansion after pinch is a dominant cooling mechanism in a capillary discharge plasma.[18] A uniform expansion has been assumed for the analytical treatment. This assumption is not bad at all because we are concerned about only a short period of time in the early expansion phase. This uniform expansion leads to a linear velocity gradient in the radial direction. The radius, temperature, and density are then expressed as follows:

$$\frac{R}{R_p} = x^{3/5}, \tag{10}$$

$$\frac{T_e}{T_{e,p}} = x^{-4/5}, \tag{11}$$

$$\frac{N_i}{N_{i,p}} = x^{-6/5}, \tag{12}$$

where $x = (1 + t/t_o)$ and t measures a time elapse from pinch. The characteristic time t_o in the expansion phase is given by

$$t_o = \frac{\sqrt{3}}{5}\frac{R_p}{\sqrt{\frac{2}{m_i}T_{e,p}(z+1)}}. \tag{13}$$

Using Eqs. (2), (5), and (7), t_o is expressed as

$$t_o \simeq 3.5\times 10^{-2}\frac{\tau_p}{\sqrt{z}}, \tag{14}$$

$$\simeq 14\frac{R_o}{z} \ [\mathrm{ns}]. \tag{15}$$

In this derivation, $z+1 \simeq z$ is used for a reasonably high-z elements and z is kept constant because in the case of CRPS x-ray lasers, the cooling rate should be faster than the recombination rate; hence, the decrease of the ionization degree should not be significant. In the long run, the recombination process will prevail, but at least during the initial period of the expansion when the population inversion occurs, the recombination process might be neglected. The validity of this assumption leads to a limitation to the atomic number z up to which the current analysis is applicable. The further discussion on this matter will be presented in the following subsection.

C. Phase of gain formation

Adequate conditions for the electron density and temperature to produce a gain on the H_α line in the H-like ion were suggested by Elton[21] to be

$$N_{e,g} = 10^{14} \ z^7 \ [\mathrm{cm}^{-3}], \tag{16}$$

$$T_{e,g} \simeq C\times \ 13.6z^2 \ [\mathrm{eV}]. \tag{17}$$

For the electron temperature, Elton used $C = 0.05$ in his analysis but $C = 0.1$ were taken here, following MHD simulation results for carbon.[14] Using Eqs. (5), (11), and (17), the time at which the population inversion occurs is estimated to be $x_g = 7.5$, which leads to the estimation of the initial density,

$$N_o = 1.1\times 10^{13} \ z^6 \ [\mathrm{cm}^{-3}], \tag{18}$$

with the help of Eqs. (8), (12), (16) and $N_{e,g} = zN_{i,g}$. The consideration of the optical trapping of the L_α line in the H-like ion gives a condition for the plasma radius. Since the radiative reabsorption of this line repopulates the lower lasing level, $n = 2$, working against the formation of a population inversion, it is required that the optical depth, κ, of this line is required to be less than unity. In the expansion phase of a capillary plasma, there is a large velocity gradient in the radial direction. The Doppler shift due to this velocity gradient greatly reduces the optical trapping effect. The optical depth, κ, taking into account the velocity gradient effect,[22,23] is written by

$$\kappa = 1.1\times 10^{-15}\epsilon\lambda N_1 f_{12}\left(\frac{2z}{T_g}\right)^{1/2} R_g \frac{1}{\left(\frac{v_g}{v_{th,g}}\right)}, \tag{19}$$

where $\lambda = 121.8/z^2$ nm, the wavelength of the L_α line of the H-like ion, and $f_{gl} = 0.4162$, the oscillator strength of the line. N_1 is the population density of the ground state of the H-like ion at the time of maximum gain, i.e., $x = x_g$, which is approximated to be $0.1\times N_{i,g}$, where the ion density, $N_{i,g}$ is given by Eq. (12). R_g is the plasma radius, v_g and $v_{g,\mathrm{th}}$ are the velocity of the expanding plasma column and the thermal velocity, respectively. v_g is obtained by taking the derivative of Eq. (10) with respect to time at $x = x_g$. ϵ is the anomalous factor which was introduced by Pert to bring a marginal agreement with experiments.[24] In this analysis, $\epsilon = 0.3$ is adopted. Then the condition of $\kappa \leqslant 1$ at $x = x_g$ leads to

$$R_g \leqslant 6.9 \; z^{-3} \; [\mathrm{cm}]. \tag{20}$$

Using Eqs. (7) and (10), we get a condition for the initial radius,

$$R_o \leqslant 20.8 \; z^{-3} \; [\mathrm{cm}]. \tag{21}$$

The scaling for a current pulse comes from the consideration of an efficient discharge. For an efficient discharge, the pinch is required to occur near the first peak of the current pulse, $\tau_p = \tau/4$. This requirement, together with Eqs. (6), (18), and (21), gives the following conditions for a discharge current pulse:

$$I_o = 6.3 \; z \; [\mathrm{kA}], \tag{22}$$

$$\tau = 3.3\times 10^4 \; z^{-7/2} \; [\mathrm{ns}]. \tag{23}$$

Now we are in the position to discuss the assumption, which we left in the previous subsection, that the degree of the ionization does not change significantly in the early period of expansion. In order for this assumption to be valid, the following inequality needs to be satisfied:

$$\frac{1}{T_e}\left|\frac{dT_e}{dt}\right| > P^{3b}, \tag{24}$$

where P^{3b} is the three-body recombination rate from a fully stripped ion to the ground state of the H-like ion. Here only the three-body recombination rate is considered because it is a dominant recombination process in a high density and low temperature plasma which is relevant to the CRPS x-ray lasers. P^{3b} has the following form:[21]

$$P^{3b} = 1.4\times 10^{-31} z^{-6} N_e^2 \left(\frac{E_{\mathrm{ion}}}{T_e}\right)^2 \exp\left[\frac{E_{\mathrm{ion}}}{4T_e}\right] \; [\mathrm{s}^{-1}], \tag{25}$$

$$= 1.7 \; z^8. \tag{26}$$

Equation (26) is obtained using Eqs. (16) and (17) at $x = x_g$. The cooling rate at $x = x_g$ is obtained from Eq. (11),

$$\frac{1}{T_e}\left|\frac{dT_e}{dt}\right| = 3.7\times 10^5 \; z^4 \; [\mathrm{s}^{-1}]. \tag{27}$$

Equations (26) and (27) tell us that the recombination rate increases faster than the cooling rate as the atomic number increases. Hence, the application of a capillary discharge to CRPS x-ray lasers is limited to the atoms with

$$z < 21. \tag{28}$$

The previous study on the effect of plasma density on the recombination process[25] revealed that in a high density plasma, there is an enhancement in the recombination due to the cascading processes following the recombination to excited levels, especially at a low temperature. In the case of carbon ion, the enhancement by a factor of 10 was observed. The inclusion of this effect by multiplying a factor of 10 to P^{3b} results in reducing the number of applicable elements,

$$z < 13. \tag{29}$$

This limit is not too strict; Eq. (29) means that for atoms with $z > 13$, the recombination from fully stripped ions to H-like ions becomes significant even during the initial period of expansion. This leads to smaller gain values, eventually to no gain as the atomic number increases further.

III. COMPARISON WITH SIMULATION AND DISCUSSION

To justify assumptions made in the derivation of the z-scaling formulas and confirm the analytical results, a series of MHD simulations was performed. The MHD code used for this purpose has been developed for the study of the dynamics of a Z-pinch plasma and its application to x-ray lasers. The detailed description on the code can be found in Refs. 18 and 25. This code requires the inputs such as the initial density, the initial radius, the peak current strength, and the duration of the current pulse. The input parameters obtained from Eqs. (18) and (21)–(23) were used for the simulations. The MHD simulations were done for the elements of $z = 4-13$. The MHD simulation results (open circles) are presented in Fig. 1 along with the analytical results (crosses) by the z-scaling formulas. The plasma parameters compared are the time of pinch (τ_p), the time of maximum gain (τ_g), the radius of plasma (R_p, R_g), the electron temperature ($T_{e,p}, T_{e,g}$), and the electron density ($N_{e,p}, N_{e,g}$), at pinch and the time of maximum gain, respectively.

First of all, it can be clearly seen that there is a good agreement between the MHD simulations and the analytical results within a factor of 2. This indicates that in conjunction with the snowplow model, the complicated plasma dynamics is well described by a shock implosion at pinch and a subsequent adiabatic expansion. For the electron temperature at pinch, a rapid equilibration between ions and electrons was assumed. The equilibration time,[26] τ_{eq}, can be also z-scaled, using the parameters at pinch adequate for CRPS x-ray lasers,

$$\tau_{\mathrm{eq}} = 1.0\times 10^3 \; z^{-4} \; [\mathrm{ns}]. \tag{30}$$

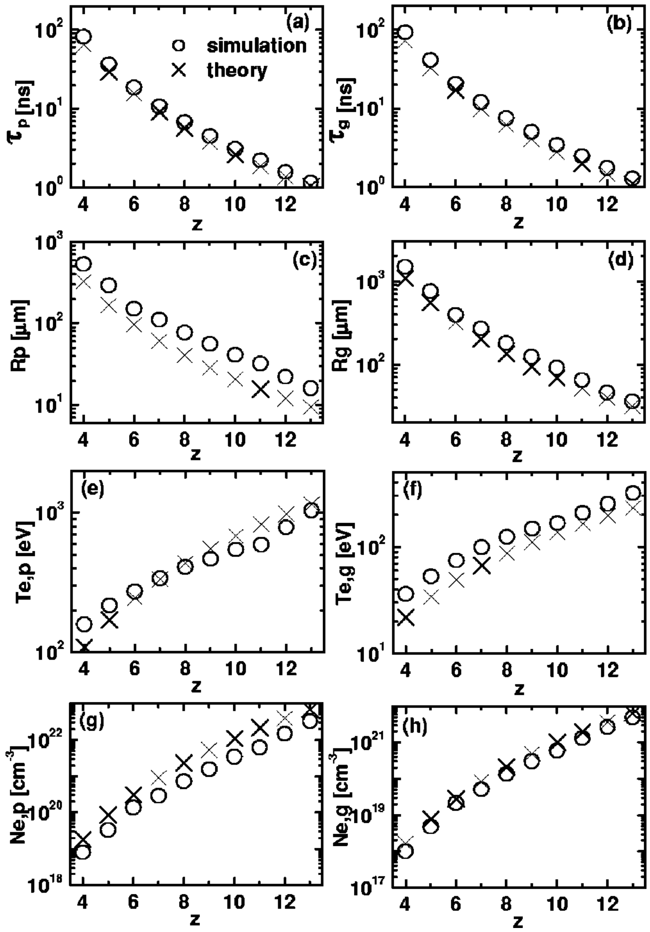

FIG. 1. The comparison between the analytic and MHD simulation results is made: (a) pinch time (τ_p), (b) the time of maximum gain (τ_g), (c) and (d) the radius of plasma (R_p,R_g), (e) and (f) the electron temperature ($T_{e,p}$,$T_{e,g}$), and (g) and (h) the electron density ($N_{e,p}$,$N_{e,g}$) at pinch and maximum gain. The MHD simulation used the input parameters given by the z-scaling formulas. The open circles (○) represent the results of MHD simulation and the cross (×) the analytical results from the z-scaling formulas.

Then the ratio of τ_{eq} to the pinch time, $\tau_p = \tau/4 = 8.3 \times 10^3 z^{-7/2}$ ns, is written by

$$\frac{\tau_{\mathrm{eq}}}{\tau_p} = 0.1 \times z^{-1/2} < 0.1, \tag{31}$$

which shows that under the conditions for CRPS x-ray lasers, the equilibration time is more than 10 times shorter than the pinch time. Since the characteristic times for the change of plasma parameters during pinch phase are much shorter than the pinch time, τ_{eq} may be comparable to the time scale of pinch dynamics. Thus the assumption for the equilibration is considered to be marginal. MHD simulations show that the ion temperature is up to 2 times higher than the electron temperature. This means that the ion temperature is underestimated using Eq. (4), which might be caused by both the neglect of an adiabatic heating and the rough estimation of the implosion velocity. However, the electron temperature is well described by Eq. (4) due to the compensation of the underestimation of the ion temperature by the slow equilibration rate.

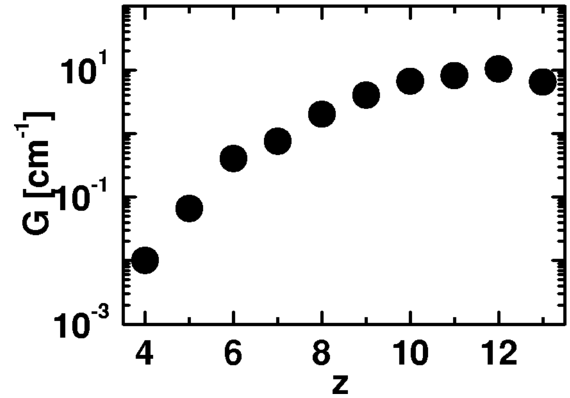

FIG. 2. Gains obtained in the MHD simulation for the parameters suggested by the z-scaling formulas.

Note that this analysis is not meant to be applied to a wide range of experimental conditions. For example, if a current is large enough to produce a pinch quite earlier than at the first peak of the current pulse, the temperature much higher than expected by this analysis could be obtained. It is claimed that the analytic formulas are valid at least for the plasma regime of CRPS x-ray lasers because (1) a good agreement was observed between the simulation and the analytical results and (2) all the MHD simulations with the inputs suggested by the z-scaling formulas produced gains as shown in Fig. 2.

Figure 2 shows the calculated maximum gains at axis. For low-z elements, the gain increases rapidly as z increases. However, the increase gets slower around z=9. The gain begins to decrease from z=13. This decrease is caused by the faster recombination process in higher-z elements. The z-scaling of gain does not follow Elton's analysis, $G \sim z^8$, because Elton did not consider the temporal evaluation of plasma parameters, which is important in capillary plasmas for CRPS x-ray lasers.

The average ionic charges, *zbar*, at pinch and at the time of maximum gain are plotted in Fig. 3. Figure 3 shows that the recombination does not take place significantly between the time of the pinch and the gain maximum, justifying the

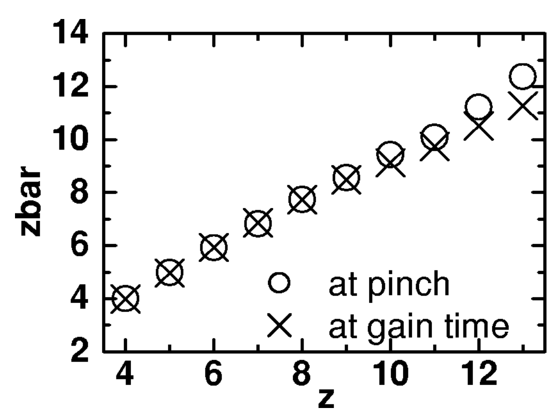

FIG. 3. The average ionic charge, *zbar*, at pinch and at the time of maximum gain obtained in the MHD simulation. The good agreement indicates that the assumption of the recombination process being negligible in the early expansion phase, used in the derivation, is valid.

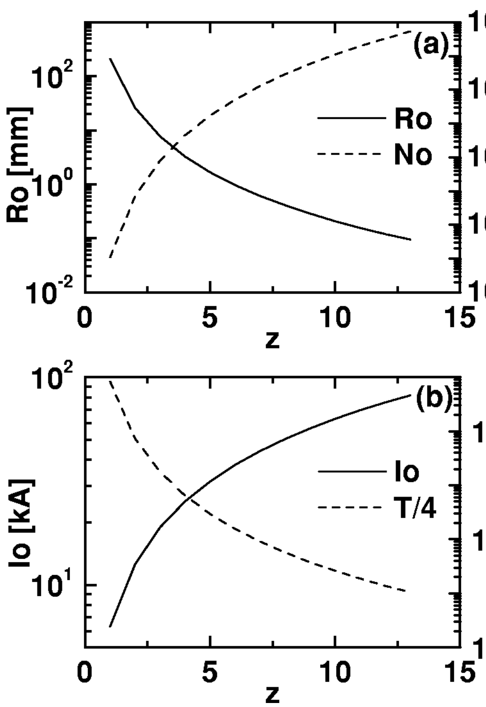

FIG. 4. Experimental parameters suggested by the z-scaling formulas are plotted with respect to z: (a) initial radius (R_o) and initial density (N_o) and (b) peak (I_o) of discharge current and its quarter-period ($\tau/4$).

assumption that during the expansion phase, the cooling rate is so fast in the early expansion phase that the change in the degree of ionization may be neglected. The discrepancy at z=12 and 13 comes from the fact that the recombination begins to be significant, since the recombination rate increases more rapidly than the cooling rate as z increases. The occurrence of the discrepancy at z=12 and 13 agrees well with the restriction condition on atomic number [Eq. (29)].

The z-scaling formulas allows us to examine the experimental parameters for CRPS x-ray lasers. The four experimental parameters obtained using Eqs. (18) and (21)–(23) are plotted in Fig. 4. The results show that as z increases or the lasing wavelength decreases, the experimental conditions become severer. The wavelength of the H_α line of Al XIII falls in the water-window region. For this element, the required experimetal parameters are $R_o = 95\ \mu$m, $N_o = 5.3 \times 10^{19}$ cm^{-3}, $I_o = 82$ kA, and $\tau/4 = 1.0$ ns. For the initial radius and density, a wire discharge[27] which is used for the generation of hard x-ray and fusion research can be employed even though the problem of stability to form a long-uniform plasma should be overcome. The condition for a current pulse is more difficult to meet. The increase of the magnitude of a peak current looks manageable but the realization of such a short pulse is hardly possible with the present high-power discharge technology.

Another application of the z-scaling formulas is to examine the electrical circuit parameters of the discharge system. The electrical circuit of a capillary discharge is considered as a RLC circuit. Since the resistance of the plasma varies in time due to the increase of the temperature and the decrease of the plasma radius, it is very difficult to analyze such a circuit with a simple formulation. Thus as a rough estimation, the plasma resistance at the pinch is assumed to be constant. For the estimation of the plasma resistance, ρ, Spitzer's resistivity,[26] which is used in our MHD simulation code, is adopted. With the plasma parameters at the pinch, the resistivity is given by

$$\rho \simeq 4.3 \times 10^{-4} l z^4 \ [\text{ohm}], \tag{32}$$

where l is the length of the plasma column in cm. In the above estimation, the Coulomb logarithm in the Spitzer's resistivity is approximated by 10. The increase of the resistance with respect to z is caused by the stronger decrease of the plasma radius, $r \propto z^{-3}$ than the increase of the temperature, $T \propto z^2$. For an efficient discharge, the condition of overdamping is used. In the derivation of the z-scaling formulas and the MHD simulation, an oscillating current pulse is assumed without damping. However, since only the first peak of the current pulse is considered in such a calculation, the overdamping condition may not alter the result if the rising current pulse is similar. Then using the conditions of current pulse [Eqs. (22) and (23)] and the plasma resistance [Eq. (32)] with the condition of overdamping, the inductance (L), the capacitance (C), the charging voltage (V_o), and the capacitor bank energy (E_B) are z-scaled as

$$L/l \simeq 1.8\ z^{1/2}\ [\text{nH}], \tag{33}$$

$$C \times l > 3.9 \times 10^7\ z^{-15/2}\ [\text{nF}], \tag{34}$$

$$V_o/l < 3.6 \times 10^{-3}\ z^5\ [\text{kV}], \tag{35}$$

$$E_B/l < 0.25\ z^{5/2}\ [\text{J}]. \tag{36}$$

For the case of aluminum plasma, the plasma length of 2.3 cm is required to reach a saturation with the gain value obtained (Fig. 2). This gives the circuit parameters as $L = 15$ nH, $C = 74$ pF, $V_o = 3.1$ MV, and $E_B = 350.4$ J. It should be noted that the currently obtained gain values are not optimized ones. Since the circuit parameters are propotional to the plasma length, the increase of the gain values directly reduces the difficulties in the construction of a discharge system. For the case of carbon plasma, the currently obtained gain of 0.4 cm^{-1} can be increased by factor of 5 with slightly different experimental parameters[14] from the ones suggested by the z-scaling formulas.

For CEPS, Shlyaptsev *et al.*[13] obtained the optimum conditions for a wavelength of $\sim$9 nm and calculated gain values through a series of MHD simulations, in which similar equations and boundary conditions were used except atomic systems. Using the z-scaling formulas, we made a comparative study of the efficiency between CRPS and CEPS. The conditions obtained by Shlyaptsev and our analysis are plotted together with respect to wavelength in Fig. 5. Around 9 nm, the initial radius and the pulse length of a current are similar but the CRPS requires a higher initial density and a lower current peak value. This suggests that for the amplification of shorter wavelengths, the CRPS may be more efficient in terms of power coupling. For gain [Fig. 5(e)], the CEPS has a higher gain than the CRPS in the long wavelength region but the CRPS has comparable or larger gains below 9 nm. An optimized condition after more refined

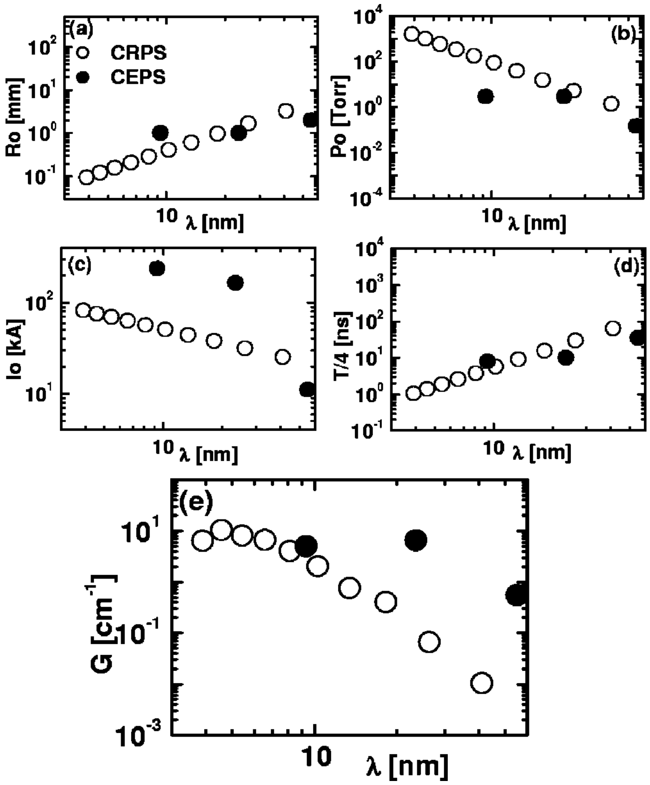

FIG. 5. The suggested experimental conditions for CRPS and CEPS are compared: (a) initial radius (R_o), (b) initial density (N_o), (c) peak current (I_o), (d) quarter-period of current pulse ($\tau/4$), and (e) gain. The data for CEPS have been taken from Ref. 13.

simulations can yield a higher gain with a less electrical power in the wavelength region shorter than 10 nm.

IV. SUMMARY

The dynamics of a capillary discharge plasma has been investigated through a simple analytical analysis, thus providing useful Z-scaling formulas for the experimental parameters of H-like CRPS x-ray lasers. The assumptions of simplified dynamics in pinch, expansion, and gain formation phases were adopted, which were justified by comparison with more detailed MHD simulations. Through this study, the key physical processes for the generation of a high gain in H-like ions produced by a capillary discharge were identified. The dominant dynamics can be well described by a shock implosion and a subsequent expansion: at pinch the temperature increases by a shock implosion with an implosion velocity of the pinch radius being divided by the pinch time. Subsequently the plasma column adiabatically expands. A high gain is produced once the expansion cooling is faster than or comparable with the recombination from fully stripped to H-like ions.

The z-scaling formulas are useful for the MHD code simulation as a guideline in search of optimum conditions, for the design of an discharge system, and for the estimation of efficiency. The analysis also shows that a high gain may be generated on the H_α line of Al XIII, which is inside the water window, in a capillary produced plasma. However, the requirement for an electrical system is rather severe and seems difficult to be realized with the present technology.

The comparison with the experimental parameters optimized for the CEPS suggests that CRPS can be a more efficient lasing scheme for the amplification in the wavelength region shorter than 10 nm with a capillary discharge. However, it needs further investigation with one simulation code to remove any discrepancy in the numerical modelings.

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