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Stepwise behavior of the core trajectory in magnetic vortex dynamics under an alternating-current magnetic field

Je-Ho Shim,1 Hong-Guang Piao,2 Sang Hyuk Lee,1 Seuh Kun Oh,1 Seong-Choo Yu,1 Seung Kee Han,1 Dong Eon Kim,3,4 and Dong-Hyun Kim1,a)

1Department of Physics, Chungbuk National University, Cheongju 361-763, South Korea
2Laboratory of Advanced Materials, Department of Materials Science and Engineering, Tsinghua University, Beijing, China
3Department of Physics and Center for Attosecond Science and Technology, POSTECH, Pohang 790-784, South Korea
4Max Planck Center for Attosecond Science, Pohang, 790-784, South Korea

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We predict that the radial distance of a magnetic vortex core from the disk center shows a stepwise behavior during initial excited motion under an alternating-current magnetic field by means of micromagnetic simulations. The stepwise behavior is clearly observed around the resonance frequency and depends on the amplitude and frequency of the external magnetic field. It has been found that the stepwise behavior originates from the relative phase difference between the gyrovector and the radial distance of the vortex core. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4803065]

I. INTRODUCTION

Recently, magnetic vortex structures have attracted much interest due to possible applications of various spintronic devices based on magnetic vortex structures. For instance, microwave generation from magnetic vortex core motion excited by a spin transfer torque is considered to be a promising technique for radio frequency generation. Another possible application is a magnetic memory scheme based on a magnetic vortex structure, as it has been reported that vortex core switching can be controlled by tuning alternating-current (AC) field frequency. Several works have been devoted to understanding the controlling mechanism of the vortex core during the core’s gyrotropic motion by a pulsed magnetic field, a horizontal AC external magnetic field, or an AC spin current.

To fully understand vortex core dynamics, not only the core switching mechanism under an AC magnetic field or AC spin current but also the transient and steady-state motion of the vortex core under AC excitation should be well understood. The understanding of core dynamics under AC excitation becomes more essential considering possible spintronic applications based on magnetic vortex structures since the devices include fast AC operation for practical applications. Very recently, it has been reported that vortex core dynamics becomes complex and nonlinear around the resonance condition under an AC magnetic field, which is an intriguing result considering the simple Thiele equation describing excited vortex core motion. The nonlinearity of the core motion has been known to be explainable by the simple dynamic correction of a gyrovector and a damping tensor in the Thiele equation. Nonlinear dynamics of the core motion driven by a spin-polarized current has been also explored with an analytical model, predicting that the nonlinearity mainly arises from magnetostatic and Zeeman energies. Although the simple dynamic correction of the gyrovector depending on a vortex core position and thus, on the magnetostatic and Zeeman energies as well, was valid in reproducing the nonlinear and complex vortex core dynamics, little is known of the detailed mechanism of the gyrovector modification for the core under an AC field or current. In this work, we report that there exists a strong correlation between the gyrovector and core radial distance from the disk center, which is predicted to exhibit stepwise behavior of the AC-forced vortex core motion.

II. MICROMAGNETIC SIMULATION AND ANALYTICAL MODEL

We have carried out micromagnetic simulations to investigate the magnetic vortex dynamics of a ferromagnetic disk under various AC magnetic fields based on the Landau-Lifshitz-Gilbert equation. The Gilbert damping constant of the Landau-Lifshitz-Gilbert equation was varied from 0.01 to 0.1, and the simulation cell size was $2 \times 2 \times 5 \text{nm}^3$. In the simulation, the material parameters of Permalloy were considered with an exchange stiffness coefficient of $13 \times 10^{-12} \text{J/m}$ and a saturation magnetization $M_s$ of $8.6 \times 10^5 \text{A/m}$. The disk thickness was $5 \text{nm}$, and the disk radius was varied from $150$ to $500 \text{nm}$. A sinusoidal AC field was applied along a certain direction on the disk plane, and the field direction was defined to be the x-axis. The amplitude of the AC external magnetic field was varied from 0.5 to $2.5 \text{mT}$. The AC frequency was varied as well, from $50$ to $250 \text{MHz}$ around the resonance frequency of each disk with a different radius. The resonance frequency was determined by analyzing the steady-state motion of the vortex core.

The Thiele’s equation is similar to an equation describing damped harmonic oscillation,

$$-\tilde{G} \times \tilde{X} - \tilde{D} \cdot \dot{\tilde{X}} + k \tilde{X} = u(\dot{\tilde{z}} \times \tilde{H}),$$

(1)
where the derivative is with respect to the time, \( \vec{G} \) is a gyrovector, \( D \) is a damping tensor, \( k \) is a stiffness coefficient, \( \vec{H} \) is an external magnetic field, and \( u \) is a constant depending on sample geometry.\(^{15} \) The gyrovector depends on the geometrical spin configuration of the disk as follows:\(^{12} \)

\[
\vec{G} = -\frac{M_s}{|\gamma|} \sin \theta (\nabla \theta \times \nabla \phi) dV, \tag{2}
\]

where \( M_s \) is the saturation magnetization, \( \gamma \) is the gyromagnetic ratio, \( \theta \) is the polar angle of local spin from the disk axis (z-axis), and \( \phi \) is the azimuth angle of local spin around the disk axis. In most cases, the gyrovector is approximated to be constant, which has been found to be invalid for vortex core motion under an AC field.\(^{12} \) It has been known that a gyrovector simply modified by considering z-component magnetization distribution on the disk works well in describing vortex core dynamics under an AC field,\(^{12} \) where the modified gyrovector (\( \Delta \vec{G} \)) is approximated to be proportional to the distance of the vortex core (\( R_{VC} \)) from the disk center, as in Eq. (3).

\[
\Delta \vec{G} = -1.6 \times 10^{-3} R_{VC} (\text{kg/m}\times s). \tag{3}
\]

### III. RESULTS AND DISCUSSION

In Fig. 1, we have plotted initial transient core trajectories with variation of the disk radius (R) from 150 to 500 nm. An AC field was applied with a fixed amplitude of 1 mT and with a different frequency (f) corresponding to a resonance frequency for each case of disk radius. R and f are denoted in this figure. The core trajectory was normalized by disk radius. It can be easily seen that the relative maximum kicking distance of the core from the disk center becomes larger for a larger disk, which is expected from the fact that the vortex structure becomes more rigid for a smaller disk.

It is interesting to note that there exists stepwise behavior in the core trajectory, in which a circular motion (solid) with a temporarily constant radius is followed by a motion with an increasing radius (open), which is again followed by a circular motion (solid) with a temporarily constant radius and so on. The trend remains clear until the magnetic core reaches the maximum radial distance but is still observable afterward. This stepwise behavior is found for all cases in the detailed gyrotropic core motion of Fig. 1. To investigate further, we have analyzed representative results of \( \Delta \vec{G} \) and \( R_{VC} \) for the case of Fig. 1 with the same simulation parameters. In Fig. 2, it is clear that there exists a strong correlation between \( R_{VC} \) and \( \Delta \vec{G} \) in all cases, which supports our approximation of simple proportionality between \( R_{VC} \) and \( \Delta \vec{G} \) with Eq. (3). Note that detectable periodic oscillations are observed both for \( R_{VC} \) and \( \Delta \vec{G} \) in all cases. Since an external field is applied along the x-axis in the present simulation, the equilibrium position is shifted along the y-axis.
The shift of the equilibrium position along the y-axis leads to an elliptic core trajectory. The elliptic core trajectory is observed throughout the entire timescale, even in the steady-state. An elliptical trajectory allows the core to pass the maximal radial distance two times when the core crosses the longer axis of the ellipse, whereas the core passes the minimal radial distance two times as well, leading to a doubled frequency compared to the external AC frequency. The

FIG. 2. $\Delta G$ and $R_{VC}$ during initial 30 ns period for the AC field frequency of resonance and the AC field amplitude of 1 mT.

FIG. 3. Initial vortex core motion for disks with different damping parameter ($\alpha$) for a disk with a radius of 250 nm. The AC field amplitude is 2 mT. Open circle represents $R_{VC}$ when the radius increases while a closed circle is represents when the radius decreases.
doubled frequency in the initial phase and the steady-state phase is confirmed to be the same. The small oscillations of $R_{VC}$ and $\Delta \tilde{G}$ seem to be a little bit off from each other, but still retaining overall proportionality.

We have also examined the effect of damping parameter change on stepwise behavior. As shown in Fig. 3, the core exhibits a clear stepwise trajectory for all cases of damping parameter ($\alpha$) ranging from 0.01 to 0.1, where the AC field amplitude has been fixed to be 2 mT and the trajectories are plotted until the core reaches the maximum radius. The kicking motion of the core seems to be significantly reduced, as expected, but there still exists a clear indication of the stepwise trajectory even in the case of $\alpha = 0.1$. Thus, the stepwise behavior of the initial core motion under an AC magnetic field, found in the present study, seems to be quite universal irrespective of damping parameters or disk radii.

To have a detailed analysis of the observed stepwise behavior, fast Fourier transform (FFT) amplitude and phase were determined. In Fig. 4, the FFT results of $R_{VC}$ (Fig. 4(a)) and $\Delta \tilde{G}$ (Fig. 4(b)) are plotted for the case of a 2 mT and 120 MHz AC field. The Gilbert damping constant is 0.01, and the disk radius is 250 nm. One of the main amplitude peaks of FFT at 10 MHz was found for all $R_{VC}$ and $\Delta \tilde{G}$, which can be ascribed to the envelope frequency ($\omega_{en}$). $\omega_{en}$ is the frequency of the amplitude envelop of the x (or y) position of

![FIG. 4. FFT amplitude (closed square) and phase (open square) of (a) $R_{VC}$ and (b) $\Delta \tilde{G}$.](image)

![FIG. 5. $\Delta \tilde{G}$ and $R_{VC}$ for the initial 80 ns period, which is categorized into three regions. $\Delta \tilde{G}$ vs. $R_{VC}$ is plotted on the bottom for three regions of increasing (bottom left), decreasing (bottom center), and decaying region (bottom right).](image)
the initial vortex core motion for a regime of transient motion before the dynamics becomes steady-state forced AC motion. It is observed that another major amplitude peak at 240 MHz exists only for \( R_{VC} \), whereas no peak can be found at this frequency for \( \Delta \tilde{G} \). The frequency originates from the doubling of the external AC field frequency (120 MHz), since the radius has squared sinusoidal x- and y-position terms and, thus, has a doubled frequency. It is interesting to note that the main two peaks of FFT are not related to any spin wave mode. It is well known that spin wave generation sensitively depends on the rate of external field increase. In the present case, an AC field with a frequency of less than 200 MHz is considered to be relatively slower than a pulse field with a rise time of sub-ns. The FFT analysis of Fig. 4 also supports our consideration that the observed stepwise core behavior is not related to spin wave behavior.

The phases of \( R_{VC} \) and \( \Delta \tilde{G} \) corresponding to the envelope frequency peak (10 MHz) are at about \(-180^\circ\) and \(0^\circ\), as shown in Fig. 4. Thus, \( R_{VC} \) is expected to follow a \( \cos(\omega_{en} t + \pi) \) phase while \( \Delta \tilde{G} \) to follow a \( \cos(\omega_{en} t) \) phase. The relative phase difference between \( R_{VC} \) and \( \Delta \tilde{G} \) lead to a linear relation, as in Eq. (3). On the other hand, the phases of \( R_{VC} \) and \( \Delta \tilde{G} \) corresponding to the doubled AC field frequency (240 MHz) are at about \(-90^\circ\) and \(270^\circ\), leading to a relative phase delay of \(2\pi\), which indicates that there is no effective phase difference. Thus, we can approximate that \( R_{VC} \) and \( \Delta \tilde{G} \) are composed of two terms, in which one follows envelope frequency \( (\omega_{en}) \) and the other follows doubled AC field frequency \( (\omega_{AC}) \). The envelope frequency terms \( R_{VC} \) and \( \Delta \tilde{G} \) have opposite phases of \( \pi \), so that the ratio between \( R_{VC} \) and \( \Delta \tilde{G} \) is negative and oscillates around the constant value in Eq. (3). The AC field frequency terms of \( R_{VC} \) and \( \Delta \tilde{G} \) have the same phases, so that the ratio between \( R_{VC} \) and \( \Delta \tilde{G} \) is positive and constant. The superposition of these two terms produces the observed stepwise behavior.

Stepwise behavior of the vortex core dynamics under an AC magnetic field is prominent only in the initial transient region. In Fig. 5, \( \Delta \tilde{G} \), \( R_{VC} \), and the ratio between the two (bottom three figures) are plotted together for the initial 80 ns. All the simulation parameters are kept the same as Fig. 4. It is interesting to note that stepwise behavior is observed not only for the initial increasing phase of \( \Delta \tilde{G} \) and \( R_{VC} \), as shown in the bottom left figure, but also for the decreasing phase as in the bottom right figure. However, stepwise behavior becomes hard to detect later than 40 ns for smaller envelope amplitude, as in the bottom right figure, which is understood based on the two contribution terms following \( \omega_{en} \) and \( \omega_{AC} \), as previously discussed.

IV. CONCLUSIONS

In conclusion, we have observed the stepwise behavior of the vortex core radial distance and the gyrovector for vortex structure under an AC magnetic field, which originates from the in-phase dynamics of \( \Delta \tilde{G} \) and \( R_{VC} \) with AC field frequency and out-of-phase dynamics with envelope frequency. We propose that vortex core radial distance can be tuned to have a specific constant value under an AC external field by controlling AC field frequency and amplitude.

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