Jpn. J. Appl. Phys. Vol. 37 (1998) pp. 2693–2698
Part 1, No. 5A, May 1998

The Effect of Radiation Trapping in Gain for Ne-like Kr

Dae-Soung KIM[1] and Dong-Eon KIM[2]
[1]*Department of OA, Youngwol Institute of Technology, Yongwol, Kangwon-do 230-800, Korea*
[2]*Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Korea*

(Received January 10, 1998; accepted for publication February 20, 1998)

The effect of radiation trapping in the atomic transitions of Ne-like Kr plasma has been investigated theoretically. First, the relative sublevel populations and gains for some specific $3s$–$3p$ transitions in the 27 levels of $2p^6$ and $2p^5 3l$ configurations were calculated in the range of electron density 10^{18}–10^{23} cm^{-3} for the electron temperature of 1 and 3 keV. Next, the influence of resonance radiation trapping was approximated by introducing the effective rates for some spontaneous emissions. Then, we recalculated the gains with these modified rates. It is noted that the effect of radiation trapping becomes significant at high electron densities and even increases the gain for some lasin transitions

KEYWORDS: atomic transition, level population, gain, opacity

1. Introduction

The occurrence of the lasing actions in some transitions between the $2p^5 3s$ and $2p^5 3p$ configurations of the Ne-like ions for proper electron densities for UV and soft X-rays has long been recognized since the suggestion by Zherikhin *et al.* in 1976.[1)] Palumbo and Elton[2)] reported the results of significant gain for $3s$–$3p$ carbon-like ionic transitions. Vinogradov and Shlyaptsev[3)] performed the calculations for the population level inversions for a number of ions between Mg III and Fe XVII, and concluded that a steady-state inversion with a gain can be achieved for a range of transitions. Feldman *et al.*[4)] extended the calculation to Ne-like Kr XXVII, and showed that a large number of Kr XXVII ions can produce a significant gain in transitions between $2p^5 3s$ and $2p^5 3p$ configurations. Rosen *et al.*[5)] predicted the integrated gain theoretically for some transitions in Ne-like Se. Many studies of laser-produced plasmas have demonstrated the amplified spontaneous emission in Ne-like ions.[6–8)] In recent years, the observation of soft X -ray amplification in a $J = 0 - 1$ transition was reported by Rocca *et al.*[9,10)]

In this paper we have performed a calculation for the relative level population and gain of the Ne-like Kr XXVII, using the similar model that Feldman *et al.*[4)] have used. The results for relative level populations and gains with electron densities in the range of 10^{18}–10^{22} cm^{-3} and electron temperatures of 1 and 3 keV show the same results as presented by Feldman *et al.*[4)] In addition to their results, we have taken into account the escape probability calculated by Zemansky[11)] to consider the opacity effect for lasing transitions. It is noted that the effect of the opacities becomes important for the high electron densities, and even increases the gain for some lasing transitions.

2. Theory and Calculation

For a Doppler-broadened spectral line in high temperature plasmas, the gain coefficient is given by

$$G = \frac{1}{8\pi}\left(\frac{M}{2\pi k T_{\text{ion}}}\right)^{1/2} A_{ul}\cdot\lambda_{ul}^3\cdot g_u\left(\frac{N^u}{g_u}-\frac{N^l}{g_l}\right), \quad (1)$$

where M is the atomic mass of the ion, k the Boltzmann constant, T_{ion} the ion temperature, λ_{ul} the wavelength of the transition between the upper u and lower l levels. g_u and g_l are the statistical weights of the levels, and N^u and N^l the number of ions per unit volume for each of the u and l level. Ion temperatures T_{ion} were chosen to be approximately equal to the electron temperatures T_{e}; however, T_{ion} may be smaller than T_{e}.

The rate equation for the populations N_i^l of the ion in its ith ionization stage is given by[12)]

$$\frac{dN_i^l}{dt} = -n_{\text{e}} I_i(l) N_i^l + \sum_u R_i^{lu} N_i^u + n_{\text{e}} Q_{i+1}(l) N_{i+1}^1 \quad (2)$$

where

$$\begin{aligned} R_k^{lu} &= n_{\text{e}} C_{ul}^e, \quad l > u, \\ &= n_{\text{e}} C_{ul}^d + A_{ul}, \quad l < u, \\ R_k^{ll} &= -\sum_{u>l}(n_{\text{e}} C_{lu}^d + A_{lu}) - \sum_{u>l} n_{\text{e}} C_{lu}^e, \end{aligned} \quad (3)$$

where n_{e} is the electron density, $C_{ul}^{\text{e(d)}}$ are the electron excitation rate coefficients(superscripts e and d refer to excitation and deexcitation, respectively). A_{ul} is the radiative transition rate from the state u to l. The terms $I_i(l)$ and $Q_{i+1}(l)$ represent the electron collisional ionization rate coefficient from the level l and the total recombination rate coefficient to the level l, respectively.

In the present calculation, we assume that the plasmas are in a steady-state equilibrium. In this steady-state equilibrium, the total ionization rate of a given ionization stage is exactly equal to the total recombination rate from the next higher ionization stage. Then, the populations of the levels are determined by solving the eq. (2) when $dtN_u^l/dt = 0$ for all the levels.[12)] The first and third terms in eq. (2) are not taken into account here, since we are only interested in the excited-level populations in a single ionization stage.

The level population may be written by

$$N^l = \frac{N^l}{N_I} N_I = m_i N_I \quad (4)$$

where N^l is the population in the level l, and N_I the total number density of Ne-like ion in all the levels. The relative level populations $m_l = N^l/N_I$ are then calculated from a set of coupled rate equations represented by following a simple matrix form

$$\boldsymbol{m} = \boldsymbol{R}^{-1}\cdot \boldsymbol{I}_{1,0} \quad (5)$$

where

$$\boldsymbol{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_n \end{pmatrix}$$

$$\boldsymbol{R} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ R^{21} & R^{22} & R^{23} & \cdots & R^{2n} \\ R^{31} & R^{32} & R^{33} & \cdots & R^{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R^{n1} & R^{n2} & R^{n3} & \cdots & R^{nn} \end{pmatrix}^{-1}$$

$$\boldsymbol{I}_{1,0} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{6}$$

In the calculation of the relative level population, we have used the atomic data presented by Feldman *et al.*[4] Then, considering the properties of the Kr plasma, the gain coefficient to the total population density of ions in all the levels can be rewritten in terms of the relative level populations as follows,

$$\frac{G}{N_I} = 1.6 \times 10^{-7} \cdot A_{ul} \cdot \lambda_{ul}^3 \cdot \frac{1}{T_{\text{ion}}^{1/2}} \cdot g_u \left(\frac{m_u}{g_u} - \frac{m_l}{g_l} \right), \tag{7}$$

where λ is in Angstrom, kT_{ion} in eV, and n_e in cm^{-3}.

The existence of the inversion is provided by the rapid radiative decay of the lower level. Thus the radiation emitted from the lower level should freely escape. In other words, the plasma must be optically thin to these resonance lines, at least in a transverse direction. If the optical depth for a transition is τ, we have

$$\tau = k(\lambda) \cdot L, \tag{8}$$

where $k(\lambda)$ is the absorption coefficient, and L the length of plasma medium. The absorption coefficient for the line radiation may be written

$$k(\lambda) = \frac{e^2 f}{m_e c^2} \left(\frac{\pi M c^2}{2kT_{\text{ion}}} \right)^{1/2} N_l \lambda, \tag{9}$$

where f is the absorption oscillator strength, m_e the electron mass, and e the electron charge. The opacity has the effect of increasing the population of the upper level. High opacity of the resonance transitions between $2p^6$ and $2p^5 3s$ configurations eventually results in the increase of the level populations in $2p^5 3s$ configuration, leading to the decrease of gains in transitions between $2p^5 3s$ and $2p^5 3p$ configurations. The escape probability method used in our calculation approximates this by the effective reduction of the radiative decay rates. In our case the transition rate of a relevant transition was replaced by the effective rate given by $E(\tau) \cdot A$, where $E(\tau)$ is the escape probability for a given τ and A the original radiative transition rate. For the escape probability $E(\tau)$ we have used the following polynomial fit to the tabulated values calculated by Zemansky:[11]

for $0 < \tau \leq 4.5$,

$$E(\tau) = 0.99900 - 0.69775\tau + 0.26653\tau^2 - 0.06191\tau^3 + 0.00811\tau^4 - 0.00045\tau^5, \tag{10}$$

and for $\tau > 4.5$,

$$E(\tau) = \frac{1}{\tau \sqrt{\pi \ln \tau}}. \tag{11}$$

3. Results and Discussion

We have considered 10 transitions for the population inversion between $2s^2 2p^5 3p$ and $2s^2 2p^5 3s$ levels. These transitions (see Fig. 1) are

A. $2p^5 3p\ ^1D_2 \rightarrow 2p^5 3s\ ^1P_1$,
B. $2p^5 3p\ ^3S_1 \rightarrow 2p^5 3s\ ^1P_1$,
C. $2p^5 3p\ ^3P_2 \rightarrow 2p^5 3s\ ^1P_1$,
D. $2p^5 3p\ ^3P_0 \rightarrow 2p^5 3s\ ^1P_1$,
E. $2p^5 3p\ ^1S_0 \rightarrow 2p^5 3s\ ^1P_1$,

and

F. $2p^5 3p\ ^3P_0 \rightarrow 2p^5 3s\ ^3P_1$,
G. $2p^5 3p\ ^1P_1 \rightarrow 2p^5 3s\ ^3P_1$,
H. $2p^5 3p\ ^3D_1 \rightarrow 2p^5 3s\ ^3P_1$,
I. $2p^5 3p\ ^3D_2 \rightarrow 2p^5 3s\ ^3P_1$,
J. $2p^5 3p\ ^1S_0 \rightarrow 2p^5 3s\ ^3P_1$,

In Fig. 2, the relative sublevel population for each of the above 12 levels is presented as a function of electron densities at the two electron temperatures of 1 and 3 keV, respectively. The populations of the six levels in the first A to E transitions are shown in Figs. 2 (a) and 2(b), and those for the six levels in the F to G transitions are shown in Figs. 2(c) and 2(d), respectively. As shown in Fig. 2, the relative sublevel populations increase with electron density and temperature due to the increase of collisional excitation rates. For a given temperature, as the electron density increases, the relative sublevel populations merge together because the collisional deexcitation rates exceed the radiative decay rates, and the populations are distributed according to the Boltzmann distribution law. Due

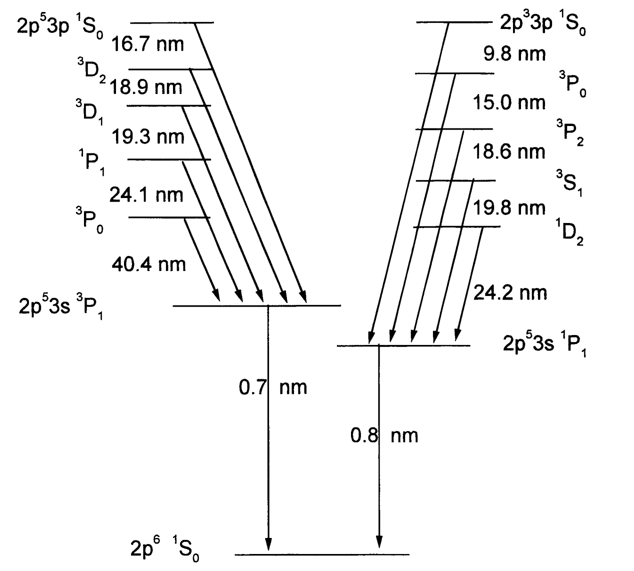

Fig. 1. A schematic of several energy levels considered for Ne-like Kr XXVII.

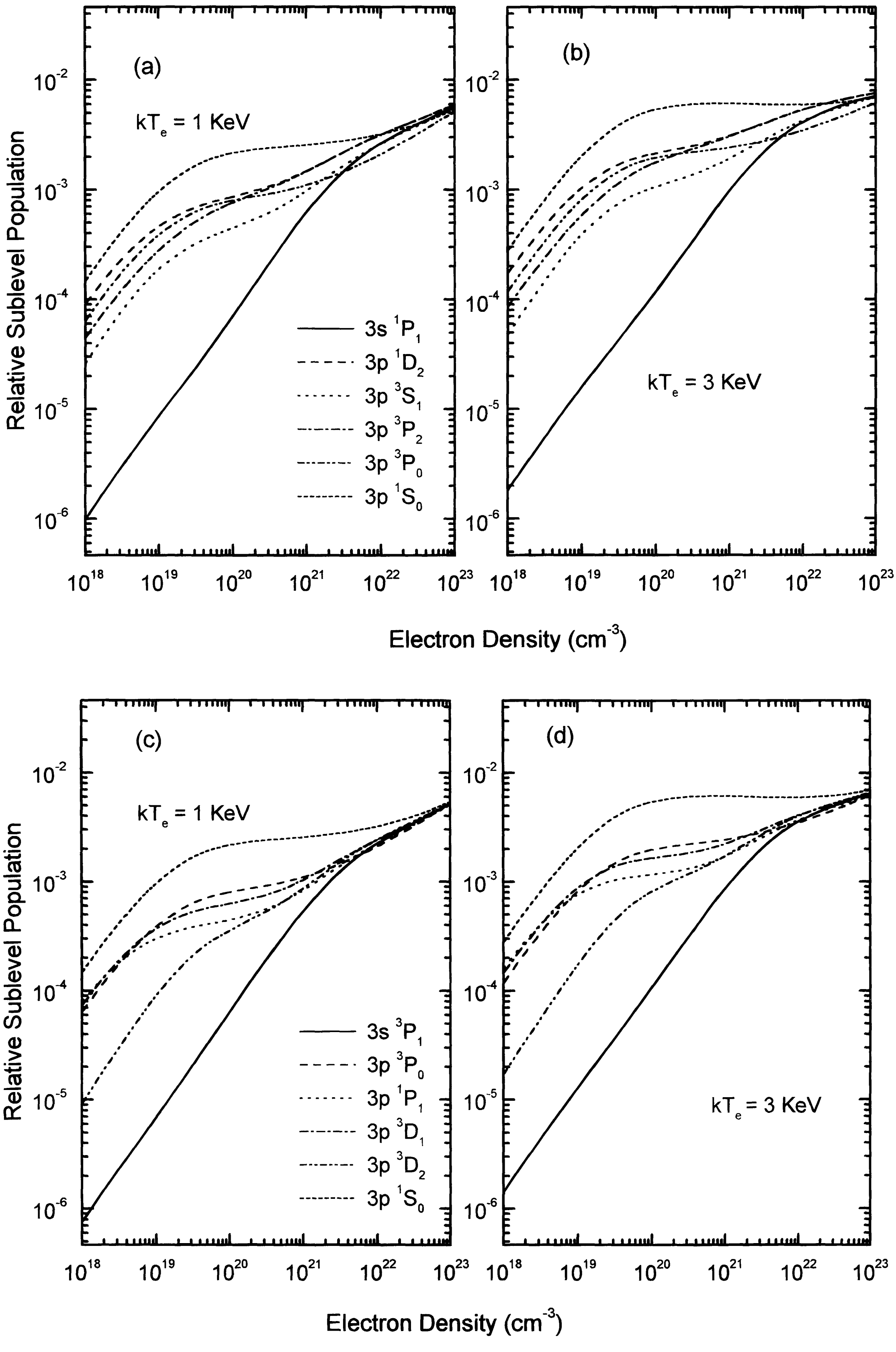

Fig. 2. Relative sublevel populations as a function of electron density for a given temperature: (a) and (c) for $T_e = 1\,\mathrm{keV}$; (b) and (d) for $T_e = 3\,\mathrm{keV}$.

to the close coupling of the $2p^5 3p\ {}^1S_0$ level to the ground level, this level is most largely populated. The population of the $3p\ {}^3P_0$ level is very close to that of the $3s\ {}^1P_1$ level, and its population inversion with respect to the $3s\ {}^1P_1$ level collapses above an electron density of $10^{21}\,\mathrm{cm}^{-3}$. This can be explained by the fact that the collisional coupling of the $3p\ {}^3P_0$ level to the ground level is not as strong as that of the $2p\ {}^1S_0$ level but the level decays radiatively faster than the other levels $3p\ ^1D_2$, $^3S_1$, and $^3P_2$.

Figure 3 shows the gains divided by the total number density of Ne-like Kr ion. For an electron density below $10^{21}\,\mathrm{cm}^{-3}$, these gains increase with the electron density due to the increase of the collisional excitation rates. As the electron density increases and reaches a value of about $10^{22}\,\mathrm{cm}^{-3}$, the gains rapidly decrease because the collisional deexcitation rates exceed the radiative decay rates and the

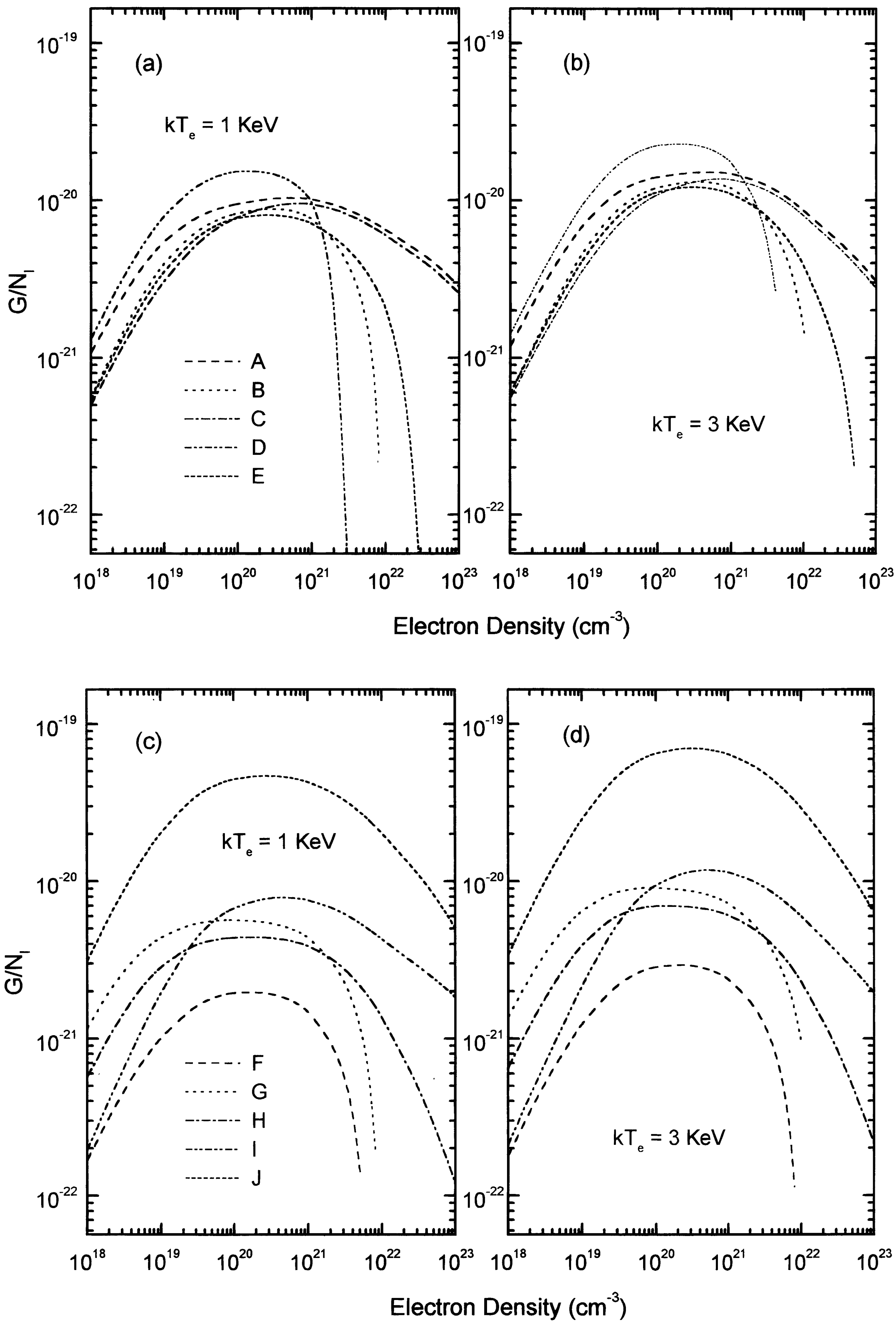

Fig. 3. Gain coefficients per ion density as a function of electron density for a given temperature: (a) and (c) for $T_e = 1$ keV; (b) and (d) for $T_e = 3$ keV. The designation for transitions are given in the text as well as in Fig. 1.

population distribution approaches a Boltzmann distribution. The gains reach their maximum in the region of $N_e = 10^{21}$ to 10^{22} cm^{-3} where the collisional deexcitation rate of each transition becomes comparable to its radiative decay rate.

We have also studied the effect of radiation trapping. In the calculation, the following transitions having larger opacities

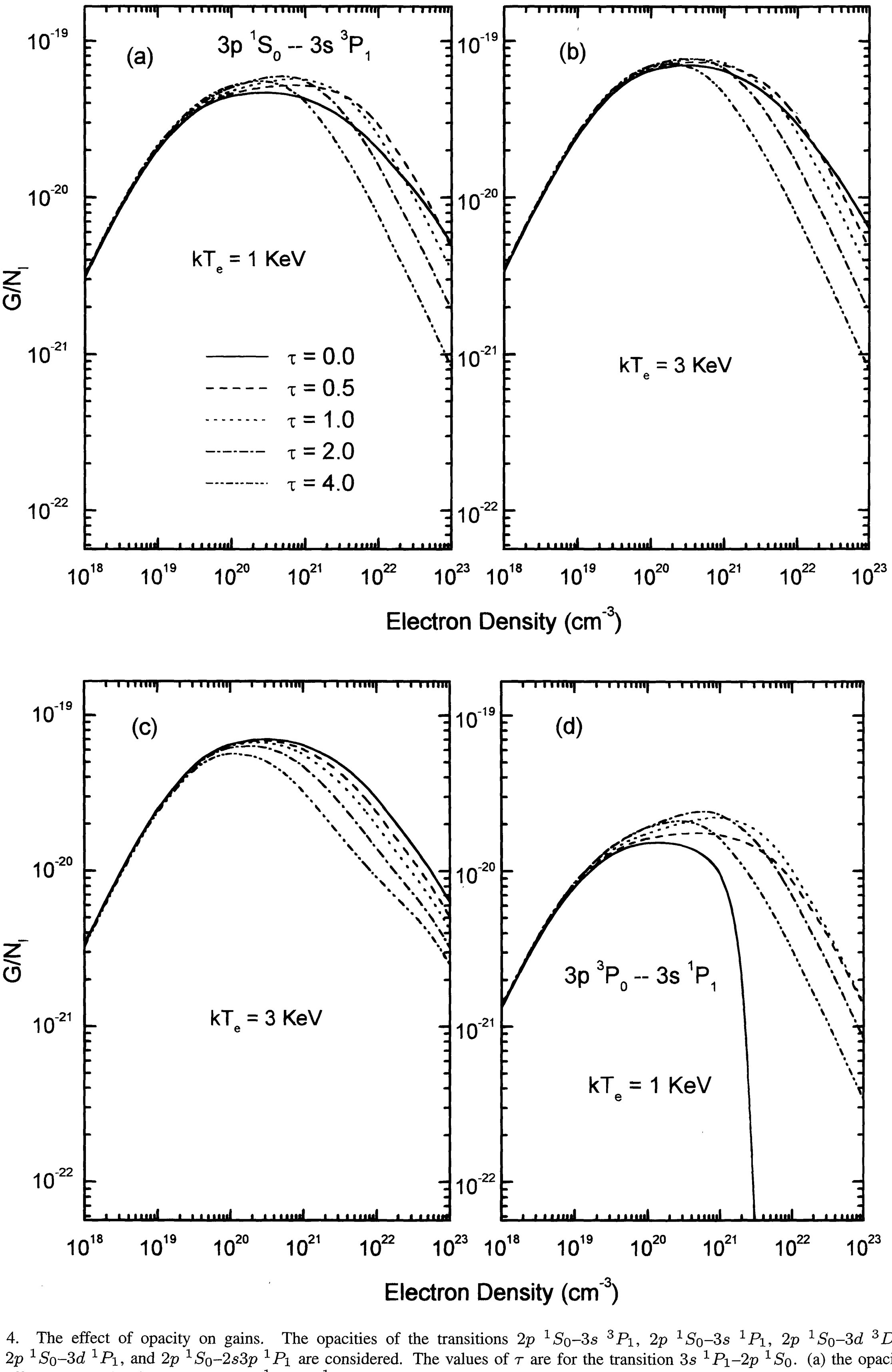

Fig. 4. The effect of opacity on gains. The opacities of the transitions $2p\ ^1S_0$–$3s\ ^3P_1$, $2p\ ^1S_0$–$3s\ ^1P_1$, $2p\ ^1S_0$–$3d\ ^3D_1$, $2p\ ^1S_0$–$3d\ ^1P_1$, and $2p\ ^1S_0$–$2s3p\ ^1P_1$ are considered. The values of τ are for the transition $3s\ ^1P_1$–$2p\ ^1S_0$. (a) the opacity effect on the gain of the transition $3p\ ^1S_0$–$3s\ ^1P_1$ for $T_e = 1$ keV, (b) the same as in (a) but with $T_e = 3$ keV, (c) the same as in (b) but in this case only the opacities of the transitions $3s\ ^1P_1$–$2p\ ^1S_0$ and $3s\ ^3P_1$–$2p\ ^1S_0$ are taken into account, (d) the opacity effect on the gain of the transition $3p\ ^3P_0$–$3s\ ^3P_1$ for $T_e = 1$ keV.

than the other transitions have been taken into consideration: $2p\ ^1S_0$–$3s\ ^1P_1$, $2p\ ^1S_0$–$3s\ ^3P_1$, $2p\ ^1S_0$–$3p\ ^3P_1$, $2p\ ^1S_0$–$3d\ ^3D_1$, and $2p\ ^1S_0$–$2p\ ^1P_1$. For a given opacity of a transition α, the opacity of a transition β can be obtained by

$$\tau_\beta = \tau_\alpha \cdot \frac{\lambda_\beta}{\lambda_\alpha} \cdot \frac{f_\beta}{f_\alpha} \tag{12}$$

which only depends on atomic properties. In the calculation the $2p\ ^1S_0$–$3s\ ^3P_1$ transition was used as a reference: the values of τ given in Fig. 4 are for this transition. The opacities of the other four transitions were calculated using eq. (12). Figure 4 shows the effect of opacity on the transitions $3p\ ^1S_0$–$3s\ ^1P_1$ and $3p\ ^3D_2$–$3s\ ^3P_1$. Below an electron density of $10^{19}\ \text{cm}^{-3}$, the opacity effect is rather small for τ up to 2. The opacity effect becomes important for electron densities larger than $10^{19}\ \text{cm}^{-3}$ or for an opacity larger than 1. For densities in the range of 10^{19}–$10^{21}\ \text{cm}^{-3}$, the opacity increases the gains. In this range of electron densities, the opacities on the transitions $2p\ ^1S_0$–$3p\ ^3P_1$, $2p\ ^1S_0$–$3p\ ^3D_1$, $2p\ ^1S_0$–$3d\ ^1P_1$, and $2p\ ^1S_0$–$2s3p\ ^1P_1$ reduce the radiative decay rates, leading to an effective increase of the population of the $3p\ ^1S_0$ level via collisional and radiative cascades. The effective gain growth is large when the opacity is small. It is noted that the gain decreases with large opacity at high electron density. When the opacities on the transitions are neglected, the gain of the $3p\ ^1S_0$–$3s\ ^1P_1$ decreases as expected as shown in Fig. 4(c). The opacity has a larger effect at low temperature than at high temperature: when the pumping rate is small as at low temperatures, even a slight reduction in the decay rate leads to a significant change in the population distribution. The opacity effect for transition $3s\ ^1P_1$–$3p\ ^3P_0$ is shown in Fig. 4(d) and shows a dramatic increase in gain at high electron densities.

4. Conclusion

We have studied the characteristics of the population of the levels in the $2s^22p^53s$ and $2p^22p^53p$ configurations and of the gains due to the fast transitions between them. The $3p\ ^3P_0$–$3s\ ^1P_1$, $3p\ ^3D_2$–$3s\ ^3P_1$ and $3p\ ^1S_0$–$3s\ ^3P_1$ transitions have large gains. The maximum gain at each of the transitions occurs around the electron density of $10^{20}\ \text{cm}^{-3}$. The gain structures at low and high temperatures are similar.

The effect on the gains due to reabsorption seems negligible if the opacity of the $2p\ ^1S_0$–$3s\ ^3P_1$ is less than 1 and the electron density is smaller than $10^{19}\ \text{cm}^{-3}$. For electron densities between 10^{19}–$10^{22}\ \text{cm}^{-3}$, the opacity increases the gains because the reduced radiative decay rates of the transitions between $2s^22p^53d$, $2s2p^63p$, and $2s^22p^6$ configurations eventually result in the increase of the population of the $2s^2sp^53p\ ^1S_0$ level via collisional and radiative cascades.

The low gain just above an electron density of $10^{21}\ \text{cm}^{-3}$ for the $3s\ ^1P_1$–$3p\ ^3P_0$ transition increase dramatically due to the opacity effect.

Acknowledgements

This work has been supported in part by Korea Research Foundation performed in the 1997 program year, POSTECH/BSRI special fund in 1997, the Basic Science Research Institute Program, Ministry of Education, 1997 (Porject No: BSRI-97-2439).

1) A. Zherikhin, K. Koshelev and V. Letokhov: Sov. J. Quantum Electron. **6** (1976) 82.
2) L. J. Palumbo and R. C. Elton: J. Opt. Soc. Am. **67** (1977) 480.
3) A. V. Vinogradov and V. N. Shlyaptsev: Sov. J. Quantum Electron. **1** (1980) 754.
4) U. Feldman, A. K. Bhatia and S. Suckewer: J. Appl. Phys. **54** (1983) 2188.
5) M. D. Rosen, P. L. Hagelstein, D. L. Matthews, E. M. Campbell, A. U. Hazi, B. L. Whitten, B. MacGowan, R. E. Turner, and R. W. Lee: Phys. Rev. Lett. **54** (1985) 106.
6) D. L. Matthews, P. L. Hagelstein, M. D. Rosen, M. J. Eckart, N. M. Ceglio, A. U. Hazi, H. Medecki, B. J. MacGowan, J. E. Trebes, B. L. Whitten, E. M. Campbell, C. W. Hatcher, A. M. Hawryluk, R. L. Kauffman, L. D. Pleasance, G. Rambach, J. H. Scofield, G. Stone and T. A. Weaver: Phys. Rev. Lett. **54** (1985). 110
7) B. J. MacGowan, M. D. Rosen, M. J. Eckart, P. L. Hagelstein, D. L. Matthews, D. G. Nilson, T. W. Phillips, J. H. Scofield, G. Shimkaveg, J. E. Trebes, R. S. Walling, B. L. Whitten and J. G. Woodworth: J. Appl. Phys. **61** (1987) 5243.
8) T. N. Lee, E. A. McLean and R. C. Elton: Phys. Rev. Lett. **59** (1987) 1185.
9) J. J. Rocca, V. Shlyaptsev, F. G. Tomasel, O. D. Cortzar, D. Hartshorn and J. L. A. Chilla: Phys. Rev. Lett. **73** (1994) 2192.
10) J. J. Rocca, F. G. Tomasel, M. C. Marconi, V. N. Shlyaptsev, J. L. A. Chilla, B. T. Szapiro and G. Giudice: Phys. Plasma **2** (1994) 2547.
11) M. W. Zemansky, Phys. Rev. **36** (1930) 219.
12) Y. T. Lee: J. Quant. Spectrosc. Radiat. Transf. **38** (1987) 131.