An autocorrelator based on a Fabry-Perot interferometer

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Abstract: An autocorrelator based on a Fabry-Perot interferometer is proposed for ultrashort pulse measurement. Main features of this autocorrelator due to the superposition of multiple pulses were investigated experimentally and theoretically. It turns out that the signal from a Fabry-Perot interferometer can be used as an autocorrelator signal. This autocorrelator provides more compact setup with a much easier alignment than a conventional autocorrelator based on a Michelson interferometer.

References and links


1. Introduction

The ultrafast optical diagnostic methods have been developed since the advent of the picosecond laser pulse. Such a pulse duration is corresponding to the limit of the response time of electrical devices and is too short for the direct measurement with electrical devices. Many methods are based on the self-referencing technique such as autocorrelation to measure ultrashort pulses down to 3.7 fs [1]. There are techniques, such as FROG [2], SPIDER [3] or MOSAIC [4] that can fully reconstruct the electric field of an optical pulse. It is now possible to measure a femtosecond optical pulse directly by using even shorter attosecond EUV pulse
However, the autocorrelator is still a convenient tool for estimating a pulse duration because of its simplicity, which does not require computing to retrieve the information. Typically, a scanning type autocorrelator consists of a Michelson interferometer and an optical parametric medium or multi-photon absorption detector to produce the autocorrelation signal. In a Michelson interferometer as shown in Fig. 1(a), a test pulse is split into two replicas and each of them is sent to two orthogonal delay arms. The pulses are reflected back at the end of arms by normal incident mirrors or retro-reflectors. Two arms should be aligned such that two optical beams are recombined in parallel. An autocorrelator using a modified Mach-Zehnder interferometer also requires two separate delay arms [6]. Previously, an in-line scanning autocorrelator is reported by swinging birefringent plate rather than translating mirrors [7]. Recently, an autocorrelator using double wedge interferometer realized the in-line configuration as a scanning type autocorrelator, but it cannot be used with a lens which is necessary for low power measurement, because the angular dispersion is introduced by the wedge pair [8].

2. Autocorrelation using a Fabry-Perot interferometer

Here, we introduce a new type of autocorrelator using a Fabry-Perot (FP) interferometer as shown in Fig. 1(b). The Fabry-Perot autocorrelator (FPAC) requires only two partially reflecting flat surfaces in parallel. Thus the optical layout is simpler and more compact than that of a Michelson autocorrelator (MAC): in the case of MAC, two arms are perpendicular to each other, while all optical components are aligned along a single optical axis in the case of FPAC. An ideal FP interferometer produces the infinite number of pulses with multiple delays due to two parallel reflecting surfaces. All of those replicas are sent to the nonlinear medium such as a BBO crystal to generate the second-harmonic like a conventional autocorrelator.

By assuming non-dispersive plane-wave propagation, the autocorrelation trace of FPAC looks similar to that of MAC as shown in Fig. 2 for a given Gaussian pulse of 30 fs full width at half maximum (FWHM) at a wavelength of 800 nm. While the autocorrelation of Gaussian pulse is Gaussian in the case of MAC (Fig. 2(a)), theoretical formulation of multiple pulse correlation (see Appendix) of Gaussian pulses is not a Gaussian (Fig. 2(b)).
Fig. 2. Autocorrelation trace for a Gaussian pulse of 30 fs FWHM at 800 nm carrier wavelength in the case of (a) a Michelson autocorrelator and (b) a Fabry-Perot autocorrelator. For the geometry of Fig. 1(c), the negative delay is not possible in real experiment. This issue is discussed later in text.

Firstly, numerical calculations have been done to retrieve a signal of FPAC. The FPAC signal usually exhibits shorter duration than the MAC signal for a given test pulse as shown in Fig. 2. The phase miss-matching among multiple pulses can be quickly increased as delay is getting far away from zero. Thus, FPAC signal, resulted from the coherent superposition of multiple pulses, decays much faster than MAC signal. In spite of multiple interferences, the proportionality between FWHM of a test pulse, $\Delta t_p$, and that of its FPAC, $\Delta t_{ac}$, is maintained. Since the ratio between $\Delta t_p$ and $\Delta t_{ac}$ is maintained, one can readily estimate the pulse duration from the autocorrelation duration by using the ratio, under the assumption on a certain pulse shape such as Gaussian or sech². The shape conversion factor $\alpha = \Delta t_{ac} / \Delta t_p$ of Gaussian pulse is about 0.73 for FPAC at $R = 50\%$ for each surface, while $\alpha$ is 1.414 for MAC [9]. The shape factor $\alpha$ depends on the reflectance of surfaces in the case of FPAC, as shown in Fig. 3(a). When the reflectance approaches to zero, the shape of a signal and the shape factor of FRAC become similar to those of MAC, because the effective number of transmitted pulses through FP interferometer decreases drastically for the lower reflectance.

Now we discuss the effect of a chirp on the autocorrelation signal of FPAC. To obtain useful information in practical situations, the chirped pulses due to the different propagation length of the material (fused silica) are compared. The modulated autocorrelation signals of chirped pulses are shown in Fig. 4(a) for the case of MAC. The maxima of the lower envelope get larger with the increase of a propagation length through a material. In the case of FPAC, there is also a similar feature on the correlation signal with respect to the propagation
length, but the effect is rather small to distinguish between the maxima of the lower envelope and the level of background. The FWHM of the correlation signal increases significantly with amount of dispersion as one see in Fig. 4(b). Thus one needs to examine more carefully on the tail of correlation signal when dealing with a chirped pulse. The simulation shows that the level of maxima increases for a lower reflectance windows in the case of FPAC.

3. The Fabry-Perot autocorrelator with three reflecting surfaces

In practice, zero delay (τ = 0) means two optical surfaces stick together. To measure autocorrelation signal, zero delay should be done in the middle of scanning the separation between them. Once a separation between two surfaces is reduced within a wavelength distance, it is difficult to separate these surfaces again due to van der Waals adhesion. Thus, one can hardly determine the peak of an autocorrelation signal by using a Fabry-Perot (FP) interferometer, so that it leads to an inaccuracy on estimating the pulse duration. It is also physically impossible to make a negative delay, even if one can exploit the symmetric property of autocorrelation only for a balanced autocorrelator. The problem of zero delay might be the major reason that a FP interferometer has not been practically used for an autocorrelator so far. Historically, a FP interferometer is implemented to an autocorrelator as a part of delay in a Mach-Zehnder interferometer [10], or two pairs of FP etalons are used to measure autocorrelation by analyzing the image of spectrogram [11].

Here, we introduce FPAC with three parallel interfaces, so that zero and negative delay can be realized without any difficulty. It is depicted in Fig. 5. The optical delay τ introduced by scanning the separation d, is equal to 2n0d/c, where n0 is the refractive index of the air and c is the speed of light. The time zero happens at n0 · d = ng · d0, where d0 is the...
thickness of a window and $n_g$ is the group index of the window. Then, one can have a sufficient space for driving a window without any conflict, depending on the thickness of windows. By assuming non-dispersive plane-wave propagation, the signal from FPAC with three interfaces is shown in Fig. 6, along for comparison with FPAC signal with two interfaces, for a given Gaussian pulse of 32 fs FWHM. In FPAC with three interfaces, trains of pulses form an infinite number of groups and each adjacent group is separated by the reference delay $\tau_0 (= 2n_g d_0 / c)$, which is far longer than the pulse duration itself. Thus, a smaller number of pulses are effectively involved in overall correlations in the case of three interfaces than the case of two interfaces, so that a signal of three-interface FPAC is broader than that of two-interface FPAC (see Appendix). The bottom level of the signal is higher than that of FPAC since a single leading pulse always adds an offset on the signal. The shape conversion factor $\alpha = \Delta \tau / \Delta \tau_p$ of Gaussian pulse is about 1.59 for FPAC with three interfaces ($R = 0.5$), while $\alpha$ is 0.73 for FPAC with two interfaces.

![Fig. 6. Autocorrelation trace from two-interface and three-interface FPAC for a given Gaussian pulse of 32 fs FWHM. R = 0.5 for all reflecting surfaces.](image)

In the case of three-interface FPAC, multiple reflections occur both in the air and in the solid optical medium; hence, one may not neglect the dispersion by a window, which causes the distortion of the pulse during propagation. To inspect the effect of dispersion on the autocorrelation signal, we have calculated the FPAC signal for a fused silica window of 2 mm thickness. The material dispersion is taken into account by applying spectral phases in the frequency domain such as $E'(\omega) = E(\omega) \cdot \exp[-ik(\omega)z]$, where $k(\omega)$ is the wavenumber calculated by Sellmeier equation for a fused silica and $z$ is the distance of propagation in the medium. The shape conversion factor $\alpha$ for the propagation through fused silica is plotted in Fig. 7, along with the result in the non-dispersive case for comparison.
Fig. 7. Change of shape conversion factor $\alpha$ with respect to the pulse duration, where $d_0 = 2$ mm, $R = 0.5$

The shape factor of the dispersive case approaches that of the non-dispersive case at long pulse duration, where the pulse broadening is small due to the narrow spectral bandwidth of the pulse. We note that the significant distortion is introduced for the pulse duration shorter than 30 fs FWHM for a thickness of 2 mm. However, by using a thinner window, three-interface FPAC geometry can still be used for the measurement of a pulse shorter than 30 fs. This kind of limitation always exists in any type of autocorrelator which contains dispersive optical elements such as a beam splitter in the case of a Michelson interferometer.

4. Experimental results

We have demonstrated FPAC with three reflecting surface ($R = 0.5$) using a pair of fused silica etalons. The thickness of each etalon is 1.7 mm. Both MAC and FPAC were measured using a Ti:Sapphire femtosecond oscillator. The output power is 220 mW at 91 MHz repetition rates. The spectrum of the laser pulse is centered at 800 nm wavelength with 40 nm bandwidth in full width at half maximum (FWHM). The transform limited pulse duration is 26 fs FWHM. The prism compressor was installed to minimize dispersion on the pulse. To see the effect of dispersion, autocorrelation was measured again by replacing the compressor with an isolator which introduces additional positive material dispersion. The results are shown in Fig. 8.

The shape of autocorrelation trace by MAC and FPAC are quite similar to each other except the level of background. The measured pulse duration by MAC is 26.7 fs. The ratio between the FWHM of MAC and FPAC is $\Delta \tau_{\text{MAC}} / \Delta \tau_{\text{MAC}} = 1.11$. The measured shape conversion factor in FPAC is 1.57, which is in good agreement with the theoretical conversion factor 1.59 for current conditions. In case of the chirped pulse, the modulation on the tails clearly indicates the existence of the chirp for both MAC and FPAC. Under the presence of chirp, the measured pulse duration by MAC is 63.0 fs in FWHM. Again, the ratio $\Delta \tau_{\text{MAC}} / \Delta \tau_{\text{MAC}} = 1.12$ for a chirped pulse, which is consistent with the transform-limited pulse case. It shows that FPAC provides the same information as a conventional autocorrelator.

The great merit of FP autocorrelator is its simplicity, allowing for easy alignment. The alignment involves only two non-iterative steps. At first, one can align the AR-coated etalon, in such a way that the reflected beam goes back along the same path of the incident laser beam. Next, one should align another etalon to make all transmitted laser beams merge together by tiling x and y axes.
5. Conclusion

In conclusion, we have demonstrated an autocorrelator based on a Fabry-Perot interferometer. The numerical simulation of the autocorrelation signal shows that there is a linear relationship between the width of the autocorrelation signal and the real pulse width, even though the value of this ratio, so called a shape conversion factor depends on the reflectance of etalons. By introducing an extra reflecting surface, the zero-delay problem in the conventional Fabry-Perot interferometer has been resolved. The laser pulse of 27 fs FWHM was successfully measured by using a Fabry-Perot interferometer for both transform limited pulse and chirped pulse. This study suggests that a Fabry-Perot interferometer can be used as an autocorrelator. The optical setup is more compact with a much simpler alignment than in the conventional autocorrelator based on a Michelson interferometer.

Appendix

The electric field of an optical pulse can be expressed as

\[ E(t) = \tilde{e}(t)e^{-i\omega t}, \]  

(1)
where $\varepsilon(t)$ is the field envelope and $\omega$ is the angular frequency corresponding to the carrier wavelength. When this pulse passes through a Fabry-Perot interferometer with two interfaces, the total electric field $E_{FP}$ is

$$E_{FP}(t, \tau) = \sum_{n=0}^{\infty} E_n(t, \tau),$$

(2)

where

$$E_n(t, \tau) = (1-R)R^n \varepsilon(t-n\tau),$$

(3)

and $R$ is the reflectance at both surfaces and $\tau$ is a delay due to the separation $d$ between two interfaces ($\tau = 2n_c d / c$). Autocorrelation signal due to the second harmonic generation is proportional to the following $G_z(\tau)$,

$$G_z(\tau) = \int \left| E_{FP}(t, \tau) \right|^2 dt$$

(4)

$$G_z(\tau) = \int \left| \sum_{n=0}^{\infty} E_n^2 + 2 \sum_{n=0}^{\infty} \sum_{m=0}^{n} E_n E_m \right| dt.$$  

(5)

By expanding the right hand side of Eq. (5), the correlation can be decomposed into three frequency components respect to $\tau$,

$$G_z(\tau) = A_0(\tau) + \text{Re}[A_1(\tau)e^{-i\omega \tau}] + \text{Re}[A_2(\tau)e^{-2i\omega \tau}].$$  

(6)

In general, the detection system of an autocorrelator acts as a low pass filter and so all the terms of the right hand side of Eq. (6) can be ignored except for the zero frequency term $A_0(\tau)$, which can be expressed as

$$A_0(\tau) = (1-R)^{1} \sum_{n=0}^{\infty} R^{4n} \varepsilon^4(t-n\tau) + 4 \sum_{n=0}^{\infty} \sum_{m=0}^{n} R^{2(n+m)} \varepsilon^2(t-n\tau) \varepsilon^2(t-m\tau) dt.$$  

(7)

The first summation in the integral of Eq. (7) represents the constant level on autocorrelation signal, while the second double summation determines the shape and the fringe of an autocorrelation signal. The difference between conventional two pulse correlation and Fabry-Perot correlation is the number of pulses that participate in correlations and the resultant is simply a summation of correlations between any possible combinations of two pulses out of multiple reflections thru Fabry-Perot interferometer. If $\varepsilon(t)$ is Gaussian shape, the convolution between $\varepsilon(t)$ and $\varepsilon(t-\tau)$ is Gaussian. However the FPAC signal is not Gaussian because the summation of Gaussian functions is not a Gaussian function in general.

Considering the FPAC which consists of three reflecting surfaces, the total electric field out of the interferometer, $E_{FP}$, is even more complicated to express in the time domain, not only because of increasing number of possible combinations of optical paths among three interfaces but also the electric field distortion from dispersion in propagation through a substrate material.

However, in the frequency domain approach, handling $E_{FP}$ becomes much easier by adopting the concept that any interferometer can be considered as a complex spectral filter. The second harmonic signal $A(\tau)$ is proportional to the square of the intensity; it can be expressed as

$$A(\tau) = \int \left| E(t, \tau) \cdot E^*(t, \tau) \right|^2 dt,$$

(8)
where the electric field from the interferometer $E(t, \tau)$ is

$$E(t, \tau) = \frac{1}{2\pi} \int E_0(\omega) \cdot \tilde{E}(\omega, \tau) \cdot e^{i\omega t} \, d\omega$$  \hspace{1cm} (9)$$

$$E_0(\omega) = \int E_0(t) \cdot e^{-i\omega t} \, dt,$$  \hspace{1cm} (10)

where $E_0(\omega)$ is the initial electric field in frequency domain and $\tilde{E}(\omega, \tau)$ is the complex amplitude transfer function as a kernel of inverse Fourier transform. By using the dynamic matrix $[12,13]$, one can derive the transfer function for each interferometer as follows:

$$\tilde{I}_M(\omega, \tau) = r \cdot t \cdot (e^{-i\omega \tau_0} - e^{-i\omega \tau})$$  \hspace{1cm} (11)$$

$$\tilde{I}_{FP2}(\omega, \tau) = t^2 \cdot e^{-i\omega \tau_0} \cdot \sqrt{1 - r^2 \cdot e^{-i\omega \tau}}$$  \hspace{1cm} (12)$$

$$\tilde{I}_{FP3}(\omega, \tau) = t^3 \cdot e^{-i\omega \tau_0} \cdot \sqrt{1 - r^2 \cdot (e^{-i\omega \tau} + e^{-i\omega \tau_0} - e^{-i\omega \tau + \tau_0})},$$  \hspace{1cm} (13)

where $\tilde{I}_M$, $\tilde{I}_{FP2}$ and $\tilde{I}_{FP3}$ are the transfer functions for a Michelson interferometer, a Fabry-Perot interferometer with two and three interfaces, respectively. $r$ and $t$ are the reflection and transmission coefficients where $|r|^2 + |t|^2 = 1$. $\omega$ is the angular frequency and $\tau$ is an optical delay, defined as $\tau = 2n_0 d/c$. $n_0$ is the refractive index of the air and $c$ is the speed of light. $d$ is the distance between a beam splitter and a delay mirror for a Michelson interferometer or it is the distance between two etalons for a Fabry-Perot interferometer, as shown in Fig. 5. $\tau_0$ is the reference delay which is defined as $\tau_0 = 2n_g d_g/c$, where $n_g$ is the group index of a reference medium, which corresponds to the substrate material of etalons for a Fabry-Perot interferometer or the air for a Michelson interferometer. $d_g$ is the thickness of an etalon for a Fabry-Perot interferometer or the distance between a beam splitter and a reference mirror for a Michelson interferometer.

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