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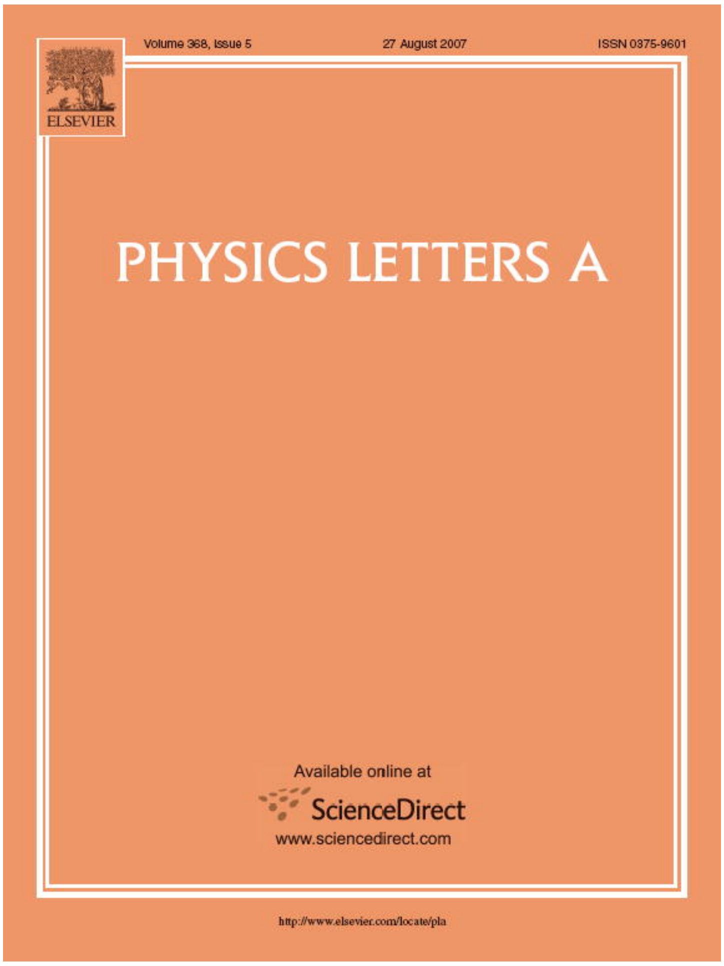

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Electron acceleration to GeV energy by a radially polarized laser

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Abstract

We present a relativistic single particle simulation of vacuum acceleration of an electron by a high-intensity radially polarized laser beam. The inherent complete symmetry of radially polarized laser beam leads to improvement in the trapping and acceleration of an electron so that an electron can be accelerated to the level of GeV. In addition, the external magnetic field further enhances the electron acceleration. Hence, an electron of ultrahigh energy was observed. The strong correlation between final electron energy and scattering angle is discussed.
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1. Introduction

The development of ultra-high power, short pulse lasers based on chirped-pulse amplification technique [1] has been intensively pursued for many years. Consequently, there have emerged many new frontier research areas. Among them, the laser particle accelerations in vacuum and plasma are very important [2–5]. The possible laser accelerations of relativistic electrons have been extensively analyzed theoretically in the past [6–13] and recently some schemes have been proposed for experimental verification [14,15]. But there are some fundamental limitations in vacuum electron accelerations. One of them is that the electric field distribution is extremely complex in the focal plane of a linearly polarized, tightly focused Gaussian beam. The significant transverse field components tend to deflect the particles and increase the beam emittance. Second, the interaction length must be limited so that the oscillatory electromagnetic field does not cancel out any net acceleration. These two problems can be overcome by using the radially polarized laser beam focused by an axicon geometry without any phase matching gas.

Polarization is one of the most important characteristics of laser radiation. In the case of linear polarization, the parameters of the beam interaction with the matter depend upon the direction of the polarization. In the case of circular polarization, these parameters are time averaged, i.e., not optimal from the viewpoint of either minimum losses or maximum absorption. In the case of radial polarization, the direction of the electrical vector in the plane of the beam cross-section is parallel to the radial direction. The focused, radially polarized, laser beam can produce a stronger longitudinal electric field than the linearly polarized beam. On another hand, if an appropriate static magnetic field is externally applied, the electron can gain significant energy. Also, it can retain considerable energy in the form of cyclotron oscillations even after passing of the laser. Hence, a magnetic field also plays a crucial role in the determination of the electron energy during acceleration in vacuum [16,17].

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Hora [2,5] has studied extensively the vacuum acceleration by nonlinear mechanism. Hartemann et al. [18] have studied the ponderomotive acceleration of electrons in vacuum by one-dimensional (1D) plane wave pulses and 2D Gaussian pulses. Malka et al. [15] have reported experimental results on MeV electron generation by the Lorentz force of an ultra-intense linearly polarized laser pulse in vacuum. They employed 1.056 μm, 300 fs, 20 J laser pulses with a focal spot diameter of 10 μm, for a peak intensity of $10^{19}\ \mathrm{W/cm^2}$ on an electron target in vacuum. The electrons with initial velocities $0.1c$ (2.5 KeV) and $0.2c$ (10 KeV) are accelerated up to 0.95 MeV and 1 MeV respectively, by a high intensity laser pulse in vacuum. Salamin [19] has shown that the lowest order axicon fields of a pettawatt (PW) laser beam, focused down to a micron size spot, can accelerate electrons from rest to GeV. Hu and Starace [20] have investigated the laser acceleration up to GeV by using highly charged ions as a source of electrons. They used 3D Monte Carlo simulations for the parameters, such as laser pulse duration, ionic charge state, and laser focusing spot size. Recently, Gupta and Suk [21,22] have introduced a frequency chirp to achieve very high electron energy during laser acceleration in vacuum.

It is well known that the cylindrical vector laser like a radially polarized beams have quite unique characteristics in comparison with linearly polarized laser. For instance, the radially polarized beams focused by a high numerical-aperture objective have been reported to have a strong longitudinal and nonpropagating electric field in the focal region. This property is thought to be suitable for optical trapping and manipulation of small particles. Furthermore, it has been predicted that a radially polarized lasers will have longitudinal electric field components with smaller spot sizes than those of linearly polarized doughnut beams. Such polarizations of the laser have been obtained by combining two linearly polarized laser output beams interferometrically or by transmitting a linear polarized laser beam through a twisted nematic liquid crystal [23–25].

In this Letter, we present the physical mechanisms involved in the generation of GeV electron beam by a high-intensity radially polarized laser. We study the direct electron acceleration by a radially polarized laser. The ponderomotive force pushes the electron in the forward direction and the electron accelerates rapidly. The electron initially at rest can gain high energy (about 450 MeV), where by following Malka et al. [15], we have employed 1 μm, 300 fs laser pulses with a focal spot size of 10 μm, for a peak intensity of about $10^{19}\ \mathrm{W/cm^2}$. If the electron is pre-accelerated (initial electron energy $\sim$1.1 MeV) then it gains about 750 MeV energy for the same experimental parameters. Second, if an axial magnetic field exists, then the betatron resonance occurs between the electron and the magnetic field. As a result, the electron can gain even higher energies. It is also observed that the electron retains the maximum energy even after passing of the laser in the presence of an external magnetic field. Hence, the magnetic field and the radial polarization of the laser enhance the energy of the accelerated electron. To find the electron beam quality, the correlation between the electron energy and the scattering angle is discussed. We examine the motion of an electron for a radially polarized laser in Section 2. We solve the coupled differential equations by using a computer simulation program to find the electron trajectory and energy in vacuum. The numerical results for electron energy, electron scattering, and radiation loss have been presented in Section 3. A brief summary of the results is presented in the last section.

2. Electron dynamics

We consider a radially polarized laser beam [3] with electric field ($\vec{E} = \hat{r}E_r + \hat{z}E_z$);

$$E_r = E_0 \frac{r}{r_0 f^2} \sin(\phi) \exp\left[-\frac{r^2}{r_0^2 f^2}\right], \tag{1}$$

$$E_z = -E_0 \frac{2}{k_0 r_0 f^2} \left[\left(1 - \frac{r^2}{r_0^2 f^2}\right) \cos(\phi) - \frac{z r^2}{R_d r_0^2 f^2} \sin(\phi)\right] \exp\left[-\frac{r^2}{r_0^2 f^2}\right], \tag{2}$$

where $\phi = k_0 z - \omega_0 t - 2\tan^{-1}(z/R_d) + zr^2/(R_d r_0^2 f^2) + \phi_0$, $f^2 = 1 + (z/R_d)^2$, $k_0 = \omega_0/c$, $R_d = k_0 r_0^2/2$ is the Rayleigh length, r_0 is the minimum laser spot size, ω_0 is the laser frequency, and c is the speed of light in vacuum. The magnetic field components related to the laser pulse can easily be deduced from Maxwell's equations. First, we discuss the case, where there is no external magnetic field applied.

Here it is worthy to discuss about the longitudinal field component of the laser. The longitudinal field component is important near the focus of the beam in vacuum. Mora and Quesnel [26] considered the effect on the electron orbits of the first-order longitudinal electric and magnetic field, and found that they lead to a nearly isotropic scattering of the particles. The polarization independence of ponderomotive force has been extensively discussed by Cicchitelli, Hora and Postle [27]. They have given an exact solution of Maxwell's equations including the longitudinal field in vacuum. On another hand, Malka et al. [15] have neglected the small longitudinal field component near the focus of the beam in vacuum. They claimed that the longitudinal field component arising from the focalization of the beam is always one order of magnitude smaller then the incident field. Hence, the longitudinal field was neglected in their calculations. But the field used by Malka et al. [15] does not satisfy the free-space Maxwell equation $\nabla \cdot \vec{E} = 0$, which gives the overestimate electron energy gain during laser acceleration in vacuum. In our case, we consider the field that makes $\nabla \cdot \vec{E} = 0$ within the paraxial approximation. This implies a survival of the finite longitudinal electric field component that can accelerate electrons traveling in the longitudinal direction.

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The equations governing electron momentum and energy are the following:

$$\frac{dp_r}{dt} = -eE_r\left(1 - \frac{\upsilon_z}{c}\right), \tag{3}$$

$$\frac{dp_z}{dt} = -e\left(E_z + \frac{\upsilon_r}{c}E_r\right), \tag{4}$$

$$\frac{d\gamma}{dt} = -e\vec{\upsilon}\cdot\vec{E}, \tag{5}$$

where, $\gamma^2 = 1 + (p_r^2 + p_z^2)/m_0^2c^2$ is the Lorentz factor, p_r and p_z are the radial and longitudinal components of the electron momentum ($\vec{p} = \hat{r}p_r + \hat{z}p_z$), υ_r and υ_z are the radial and longitudinal components of the electron velocity ($\vec{\upsilon} = \hat{r}\upsilon_r + \hat{z}\upsilon_z$), and $-e$ and m_0 are the electron's charge and rest mass, respectively.

Simulations are carried out by solving the momentum and energy equations using a relativistic single particle computer program developed by MATHEMATICA® [28]. The electron energy γ as a function of distance z is obtained for different parameters by assuming the initial electron energy γ_0. Throughout this Letter, time and length are normalized by $1/\omega_0$ and c/ω_0, respectively. Velocity, momentum, and energy are normalized by c, m_0c, and m_0c^2, respectively. The normalized laser intensity parameter is expressed as $a_0 = eE_0/m_0\omega_0c$. In all simulations below, we set parameters, $a_0 = 3$ (corresponding to laser intensity $I \sim 1.25 \times 10^{19}$ W/cm^2 and wavelength $\lambda_0 \sim 1$ μm), $a_0 = 5$ (corresponding to laser intensity $I \sim 3.46 \times 10^{19}$ W/cm^2 and wavelength $\lambda_0 \sim 1$ μm), normalized beam radius at focus $r_0' = 75, 100$ (corresponding to laser spot sizes $r_0 \sim 8$ μm, 10 μm, respectively), and $\gamma_0 = 1, 2.23$. We also use the parameters of a petawatt (PW) laser to find out the corresponding electron energy gain during acceleration.

3. Results of numerical simulations

3.1. Electron energy estimations

The electron trajectories in r–z plane are shown in Fig. 1(a) for different laser intensities $a_0 = 3$ (dotted line) and 5 (solid line). The electron is at rest ($\gamma_0 = 1$) in this case. The electron experiences a ponderomotive force due to the laser pulse. The longitudinal momentum of the electron increases due to the laser fields and the electron escapes from the laser. On axis, the electron experiences force only by the longitudinal component of the electric field of the laser, since the radial component of the electric field of the laser vanishes at all points on the longitudinal axis. As a result, initially, the electron rotates around the direction of the propagation of the laser because of the low propagation velocity. As the electron velocity approaches to the velocity of light, it moves from the axis due to the ponderomotive scattering. The enhancement in electron momentum of the electron is observed for higher laser intensity. As a result of high intensity of the laser, the electron is pushed beyond the Rayleigh distance. Far from the Rayleigh length, the electron moves freely with a speed close to the speed of light, because the laser intensity is weak due to large beam size far from the focus. Fig. 1(b) shows the energy gain during acceleration of the electron for the same numerical parameters used in Fig. 1(a). The electron accelerates to high energy by a radially polarized laser pulse. It is observed that the electron gains energy about 200 MeV and 400 MeV for laser intensities $a_0 = 3$ (dotted line) and 5 (solid line), respectively, even though the electron is initially at rest. However, in an experiment, Malka et al. [15] have reported 0.95 MeV and 1 MeV electron energy for initial electron velocities $\upsilon_{z0} = 0.1c$ and $\upsilon_{z0} = 0.2c$, respectively. The significant enhancement in electron energy is due to the fact that the radial field vanishes at axis and surviving only longitudinal field, which accelerates the electron axially. On another hand, the accelerated electron tends to get out of phase with the field. Therefore, it is decelerated and lost its energy, being left at the end

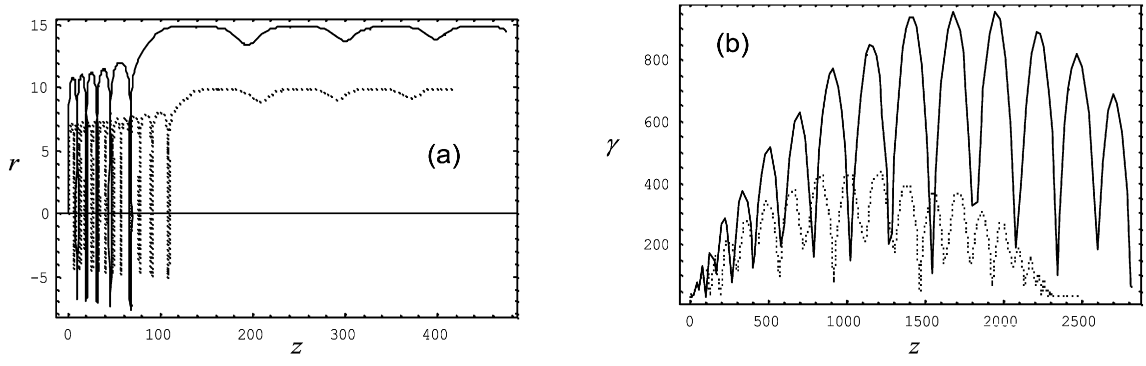

Fig. 1. (a) Trajectory of a rest electron ($\gamma_0 = 1$) in the r–z plane, (b) corresponding electron energy with distance for $a_0 = 3$ (dotted line) and $a_0 = 5$ (solid line). The other normalized parameters are $r_0' = 75$ and 100 corresponding to the laser intensities $a_0 = 3$ and 5, respectively.

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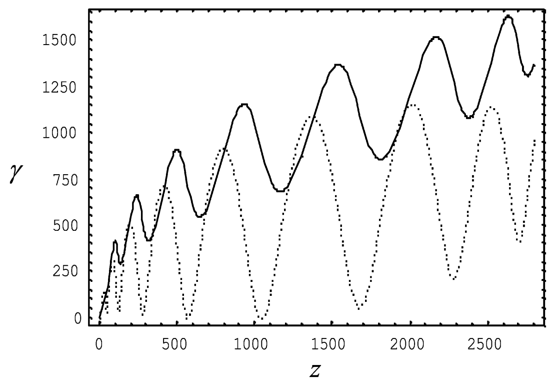

Fig. 2. Electron energy with distance for initial electron energy $\gamma_0 = 2.23$ at different laser intensities $a_0 = 3$ (dotted line) and $a_0 = 5$ (solid line). The other parameters are the same as those of Fig. 1.

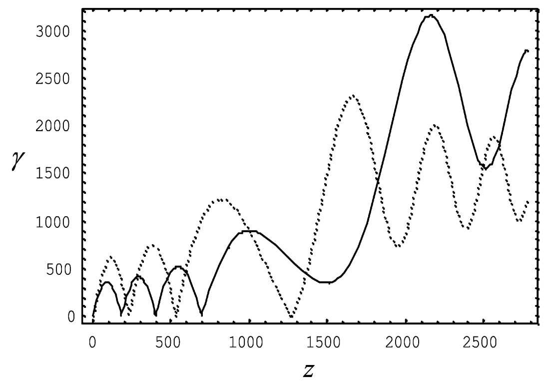

Fig. 3. Electron energy with distance in the presence of a magnetic field $b = 0.01$ ($B_0 = 1$ MG) for laser intensity $a_0 = 5$ at different initial electron energies $\gamma_0 = 2.23$ (solid line) and $\gamma_0 = 1$ (dashed line).

with a small energy of about 25 MeV for laser intensity $a_0 = 5$. However, the net electron energy gain can be enhanced by using a very high-intensity laser. In that case, due to the ponderomotive scattering, the rest electron will leave the interaction region before being decelerated and can retain significant energy as predicted by Hu and Starace [20]. Hu and Starace [20] have observed about 100 MeV retainable energy of a rest electron accelerated by a linearly polarized laser of peak intensity of $2 \times 10^{22}\ \mathrm{W\,cm^{-2}}$ ($a_0 \approx 100$), a wavelength of 1054 nm, and a beam waist in the focal region of 10 µm. If we employ a radially polarized laser of such a high intensity, our equations will yield much larger retainable electron energy than that was observed by Hu and Starace [20]. The reason behind this prediction is that in the case of linearly polarized laser, the electron loses its energy during acceleration because of the transverse deflection by the transverse field component of the laser.

In Fig. 2, we estimate the electron energy gain (γ) against the longitudinal distance for initial electron energy $\gamma_0 = 2.23$ at different laser intensities $a_0 = 3$ (dotted line) and $a_0 = 5$ (solid line). The net energy gain by the electron during acceleration is sensitive to the laser intensity and the initial electron energy. The electron can gain very high energy, if it is pre-accelerated, because the duration of interaction between the laser and the electron increases with initial electron energy. The retainable electron energy of the accelerated electron is also enhanced in this case.

The effect of an external magnetic field on maximum energy gain is also investigated. This can be estimated by simply adding the additional force imposed by the axial magnetic field $\vec{B}_0$ in the equation of motion. The effect of the imposed magnetic field on electron energy gain during acceleration by a radially polarized laser is shown in Fig. 3. The electron initially at rest gains about 1 GeV for laser intensity parameter $a_0 = 5$ and magnetic field strength parameter of $b_0 = 0.01$ (where $b_0 = eB_0/m_0c\omega_0$) (Fig. 3, dotted line). We can estimate the value of the magnetic field directly by the expression $B_0 = 2\pi m_0 c b_0/e\lambda_0$. For one micron wavelength of the laser pulse, the value of the magnetic field turns out to be about one mega gauss for $b_0 = 0.01$. This strength of the magnetic field is available nowadays in laboratories and are generated during laser-plasma interaction. If such a strong magnetic field is not available at a laboratory, then an experiment can be carried out at high initial electron energy and high laser intensity because the optimum value of the magnetic field decreases with laser intensity and initial electron energy [29]. The external magnetic field enhances the strength of $\vec{\upsilon} \times \vec{B}$ force. Therefore, the electron traverses more distance in the direction of the propagation of the laser. When the cyclotron frequency of the electron motion in the uniform magnetic field approaches to the Doppler-shifted laser frequency then the energy transfer from laser to electron will be maximum. Therefore, for an optimum magnetic field, the electron energy gain is maximized due to the resonance. Also, the static magnetic field bends the electron out of the laser path. Hence, the electron leaves the interaction region and it does not loose its energy. The electron retains considerable energy due to the effect of a magnetic field in the form of cyclotron oscillations even passing of the laser.

If the electron is pre-accelerated (has enough initial kinetic energy) then it can be accelerated to GeV energy as shown in Fig. 3 (solid line). For an initial electron energy of $\gamma_0 = 2.23$, the laser intensity parameter of $a_0 = 5$, and the magnetic field of $b_0 = 0.01$, the electron gains about 1.5 GeV energy during acceleration by a radially polarized laser. This energy gain is also retainable in this case. For a pettawatt (PW) laser, our equations may yield about 5 GeV electron energy with a suitable magnetic field. Such field intensity may be available in near future.

3.2. Electron scattering and radiation loss

A correlation between electron energy gain (γ) and scattering angle (θ) can be predicted by using the below theoretical law [30],

$$\theta(\gamma) \approx \arctan\left(\frac{\sqrt{2(\gamma/\gamma_0 - 1)/(1 + \upsilon_0/c)}}{\gamma - \gamma_0(1 - \upsilon_0/c)}\right). \tag{6}$$

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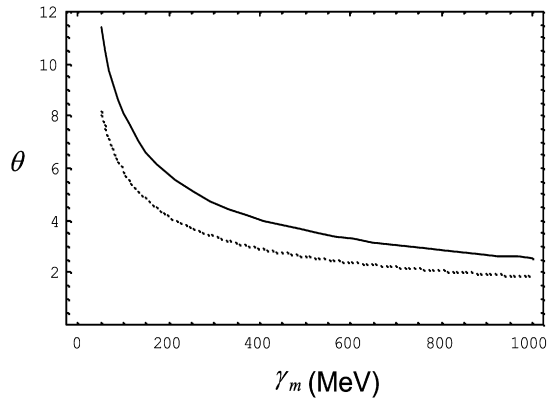

Fig. 4. Electron scattering angle (θ, in degree) with maximum electron energy (γ_m, in MeV) for laser intensity $a_0 = 5$ at different initial electron energies $\gamma_0 = 2.23$ (dashed line) and $\gamma_0 = 1$ (solid line).

From Eq. (6), we can compute the electron scattering angle corresponding to the maximum electron energy gain for different initial electron energies, where the maximum electron energy gain can be estimated from the previous equations of this Letter. Fig. 4 shows the electron scattering angle (θ, in degree) as a function of maximum kinetic energy of the electron (γ_m, in MeV) for different initial electron energies $\gamma_0 = 1$ (solid line) and $\gamma_0 = 2.23$ (dotted line), where the used laser intensity parameter is $a_0 = 5$. It is observed that the electrons with different initial velocities those have been gained different maximum energies scatter at different angles. For example, the electron of zero initial energy deflects by an angle of about 4 degree with final energy of 400 MeV, where the laser of intensity $a_0 = 5$ is operated. If the electron has enough initial energy (pre-accelerated), then the scattering angle will be reduced. This can be understood by the ponderomotive scattering during laser electron acceleration. At high-intensity of the laser, the quiver amplitude of the electron exceeds the laser spot size and it escapes from the pulse around the focus with enough kinetic energy. We found that if a high-intensity laser with a radially polarization is used for electron acceleration in vacuum, then the electron that is close to the propagation axis can be accelerated in the forward direction with small scattering.

From Fig. 1(b), it can be observed that the electron absorbs maximum energy from the field and loses it to zero after passing of the laser. The electron loses substantial energy during deceleration via radiations. The radiative loss during deceleration can be estimated by the following expression of the radiated power [30]:

$$P(t) = \frac{2e^2\gamma^6}{3c}\left[\left(\frac{d(\vec{v})}{dt}\right)^2 - \left(\vec{v} \times \frac{d\vec{v}}{dt}\right)^2\right]. \tag{7}$$

The inspection of above expression shows that the radiated power strongly depends on electron quiver velocity. For the numerical parameters used in Fig. 1(b), the radiation loss ($\Delta W \sim P \times \Delta t$) is very large, where Δt is the interaction duration. Hence, the electron cannot retain its maximum energy. Almost gained energy was loosed via radiations. If the electron is pre-accelerated then it can retain considerable energy and radiation loses are not much. This kind of energy loss may be found in linear accelerators.

4. Conclusions

We observed the acceleration of free electron in vacuum to GeV kinetic energy by a radially polarized laser. We utilize the unique properties of a radially polarized laser for electron acceleration in vacuum. The ponderomotive force due to the laser pushes the electron in forward direction. The significant enhancement in electron energy is due to fact that the radial field vanishes on axis, but only longitudinal field survives which accelerates the electron axially. The electron gains high energy, even though it is initially at rest. An electron at rest that is accelerated to relativistic energy tends to get out of phase with the field. Therefore, few MeV net energy was retained by a rest electron. However, for very high laser intensity, the rest electron can retain higher energy. Because the electron leaves the interaction region before being decelerated. Second, if an optimum magnetic field is applied during the trailing part of the laser pulse, the electron can gain and can retain significant energy in the form of cyclotron oscillations.

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